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Handbook for Automotive Reliability – Assessment and Validation.

Société des Ingénieurs de l'Automobile
Association reconnue d'utilité publique

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Associated documents:

- Glossary of a shared reliability language (in French and English, Reference DC-02)
- French version of the guide: Guide d'aide à l'estimation et à la validation de la fiabilité automobile (Reference DC-01)
- Interview survey results (reference DC-03)

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Reliability is, after all, engineering in its most practical form,

James R. Schlesinger (1929-2014) – Former United Sates Secretary of Defense.

FOREWORD

The French automotive industry is developing towards international markets and is adapting to new constraints (service contract, leasing, and extension of warranty periods).

Currently, warranty costs vary, for American car manufacturers and suppliers between 1 to 3% of their turnover[1].

Controlling the reliability during the warranty period and beyond, has significant issues, not only financially but also regarding brand image. The assessment of reliability at high mileage is a major challenge for the preparation of warranty extensions and competitiveness.

As a result, engineering offices and technical teams need to have tools and methods to provide quantified and credible assessment of the future reliability of the vehicles with a known uncertainty.

Proven methods have been developed to predict the upstream reliability without waiting for end–user feedback. These methods are the topic of this handbook and allow to quantify future reliability through validation plans involving tests and calculations. They are based on multiple data such as customer feedback, materials data, mission profiles...

The effective use of these methods faces a difficulty: the actors of the automotive industry are spending a lot of energy collecting reliability data and understanding each other's expectations.

It therefore appeared necessary to create a reference handbook for designers to build reliability in a more efficient way.

This handbook will also be a base for many industry stakeholders (manufacturers, suppliers, specifiers, designers, test specialists, calculation engineers, RAMS engineers, purchasers) to design and validate reliable products in a collective and efficient way.

To clearly identify the current difficulties and expectations, a series of interviews was conducted in 2015 with designers or specialists from 6 companies: VOLVO TRUCKS, RENAULT, HUTCHINSON, PSA, VALEO and CONTINENTAL. The results of the 37 interviews are detailed in the SIA document DC-03-01.

These difficulties can be grouped into 6 main themes:

1. Lack of knowledge and misunderstanding regarding reliability vocabulary and methodology

2. Data availability (insufficient input data, lack of experience feedback regarding customers' reliability)

3. Cost of tests and deadlines (shorter and shorter project schedule, availability of test benches)

4. Lack of resources to conduct studies and to train new people

5. Collaborative work between suppliers and manufacturers (issues related to expertise and responsibility)

6. Management: reliability should not be an option or be considered only in the case of crisis

The handbook answers some difficulties mentioned in these interviews, especially Themes 1, 2 and 3.
The handbook and the glossary's aims are to provide a base to any person leaning on reliability in their activities in order to understand the reliability vocabulary and the different phases required to build and use reliability validation plans.

[1] Reference: Warranty week: European Auto Warranty Report

This handbook introduces a methodology that defines the different phases required to quantify the "just sufficient" reliability of components.

At the end of this handbook, the reader will have understood the input and output data of each phase, as well as the most useful reliability methods by using digital and physical simulation. For simplicity and brevity, this handbook only presents the main techniques that can help manufacturing a reliable product.

It is intended for engineers / designers of the automotive industry, liable to manage reliability, to assess it, or to build the associated validation plans.

TABLE OF CONTENTS

GLOSSARY OF A SHARED RELIABILITY LANGUAGE

A glossary built by the same SIA working group comes with this handbook. It is referenced SIA DC-02. It introduces the common terms related to reliability.

Glossary terms used in the handbook are written and underlined in green.

LIST OF PRACTICAL SHEETS

LIST OF ANNEXES

PART A. METHODOLOGY

1 STRUCTURE OF THE HANDBOOK

This handbook introduces a general approach to understand how a reliability validation plan is built and used without being a reliability expert (some knowledge of statistics is still necessary). It provides a 6-phase methodology developed in **Part A.5** that remains accessible to reliability neophytes.

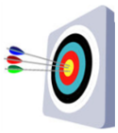
1.1 DEFINITIONS AND PRACTICAL SHEETS

This handbook is completed by a glossary giving the shared terms used in the reliability field. It is referenced SIA DC-02. Glossary terms used in this handbook are written and underlined in the text in green.

The practical sheets are indicated **in blue** in the text. Their difficulty level is rated from 1 star (easy) to 3 stars (hard).

1.2 PICTOGRAMS

This handbook is marked by icons, in order to draw the reader's attention to some part of the text. The meaning of these icons is indicated on the table below.

Icon	Meaning
	This icon indicates a KEY point, necessary information.
	This icon indicates about a TRAP to AVOID.
	This icon indicates a GOOD PRACTICE, a GOOD IDEA.

1.3 NOTATIONS AND ACRONYMS

$f(x)$	Probability density function
$f_R(x)$, $f_C(x)$	Probability density function of the strength variable R and of the stress variable C
$F(x)$	Cumulative distribution function
$F_R(x)$, $F_C(x)$	Cumulative distribution function of the strength variable R and of the stress variable C
$F^{-1}(x)$	Inverse distribution function
$R(x)$	Reliability function = $1-F(x)$ where $F(x)$ depicts the variability of a reliability parameter (number of cycles, time, stress level…)
$E[X]$	Expected value of the random variable X
$Var(X)$	Variance of the random variable X
CV_X	Coefficient of variation of the random variable X
Φ	Cumulative distribution function of the standard normal variable
μ_X	Mean of the normal random variable X
σ_X	Standard deviation of the normal random variable X
μ_{lnX}	Mean of the normal random variable $\ln(X)$, (X is a lognormal random variable)
σ_{lnX}	Standard deviation of the random normal variable $\ln(X)$, then X follows a lognormal distribution
β	Shape parameter of the Weibull distribution
η	Scale parameter of the Weibull distribution
γ	Location parameter of the Weibull distribution
λ	Failure rate of the exponential distribution
N	Number of cycles, occurrences, activations, tested components
b, B	Coefficient and constant of the acceleration model
C	Stress random variable in the Stress-Strength method
R	Strength random variable in the Stress-Strength method
A	Reference period
P_f	Customer failure probability
τ	Duration of a time-censored test (time, number of cycles, number of activations …)
k	Number of failures during a time-censored test
θ_{REX}	Distribution parameter known from feedback

θ_{mini}	Minimum value of the unknown distribution parameter for which the reliability target is met (resolution of the inverse Stress-Strength problem)
δ_c	Proportion of failures at the end of a time-censored test (test failure probability) with a confidence level c (also called test failure probability)
c	Confidence level
CR	Customer risk
SR	Supplier risk
FA	Functional analysis
HARA	Hazard Analysis And Risk Assessment
FMECA	Failure Modes, Effects and Criticality Analysis
FT	Fault Tree
HE	Hazard Event
HALT	Highly Accelerated Life Testing
ASIL	Automotive Safety Integrity Level
REX	Knowledge from the field data.
RPN	Risk Priority Number

2 INTRODUCTION

In order to design a reliable product, it must first be accepted that "0 fault" does not exist and that the product will be designed with a customer failure probability in line with the allocated reliability target. This objective is specific to each product and each company.

Reliability is not achieved by chance but is built from the beginning of the product life cycle.

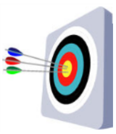
Reliability consists in controlling the risk of failure by:

- assessing the failure probability through a validation plan (analysis / calculation / test activities),
- checking that the failure probability meets the reliability target associated with the customer risk,
- optimizing the design and verification / validation cost based on the estimated reliability.

Reliability is a design engineering discipline which applies scientific knowledge to ensure a product will perform its intended function for the required duration within a given environment[2].

From a mathematical point of view, the product reliability is the probability that the product will not fail during a period of time (reference period) and under given functional conditions and environment.

Figure 1 depicts the reliability R(t) and failure functions F(t) (F(t)=1- R(t)) between t = 0 and t, of a brake caliper, in terms of the number of stress cycles. Reliability is 10% for 700 cycles meaning that the failure probability is 90% for 700 cycles. Reliability decreases with time while the failure probability increases.



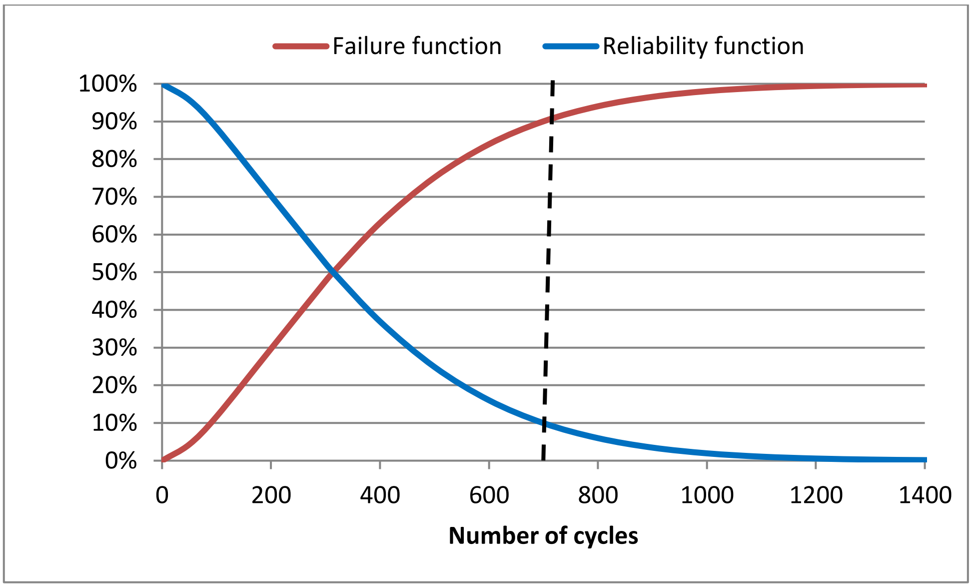


Figure 1: Reliability and failure functions.

[2] Source: IEEE Reliability Society 2006.

3 RELIABILITY ISSUES

The service life of a component is usually composed of three main phases characterized by specific instantaneous failure rates λ (failure probability, on a time interval dt, knowing that the device has worked well until time t) (*see* ***Figure 2***):

- **The period of early failures**, also called infant mortality period, characterized by an instantaneous failure rate decreasing with operating time or number of requests. These failures often root from a poor process or assembly, *e.g*: excessive scatter, drift or incidents. Usage condition and particular environment may also cause early failure. The probability of occurrence of these failures decreases with operating time of the vehicle until the entire weak population has failed. This weak population can be removed with a burn-in process in some cases.
- **The period of random failures**, also called period of useful life, characterized by a constant instantaneous failure rate. In this period, mortality (failure) is random and accidental. The randomness is due to a significant number of causes or failure modes. These failures occur while the vehicle is operating and it is generally considered that the failure probability remains the same regardless of time. This implies that the probability of failing tomorrow is the same as yesterday. This period is virtually nonexistent for mechanical devices, unlike electronic components.
- **The period of wear-out failures**, characterized by an instantaneous failure rate increasing with time or number of requests. These failures are caused by the degradation over time of the material characteristics. This degradation is related to physicochemical, mechanical phenomena... such as wear, fatigue, corrosion. It corresponds to the increase of damage in the component when the vehicle is in service.

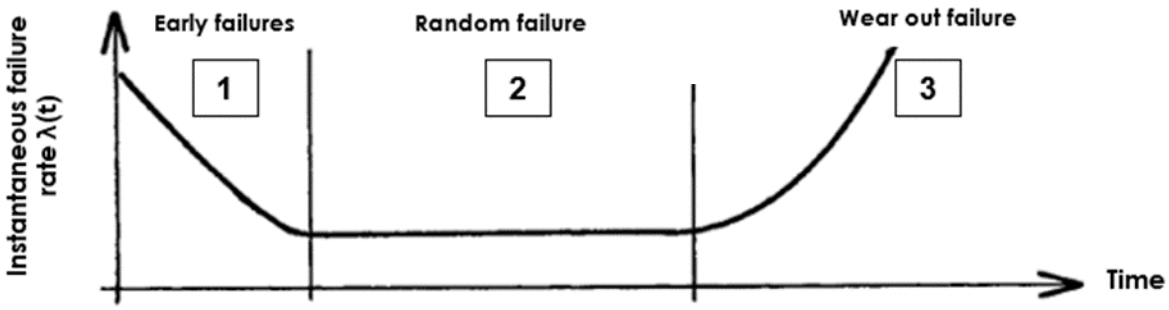


Figure 2: Instantaneous failure rate λ of a component in terms of operating time.

The evolution of the instantaneous failure rate depends on the failure type (see ***Figure 3***):

- For early failures, (zone 1) with a decreasing failure rate,
- For random failures(zone 2) with a constant failure rate,
- For wear-out failures, (zone 3) with an increasing failure rate.

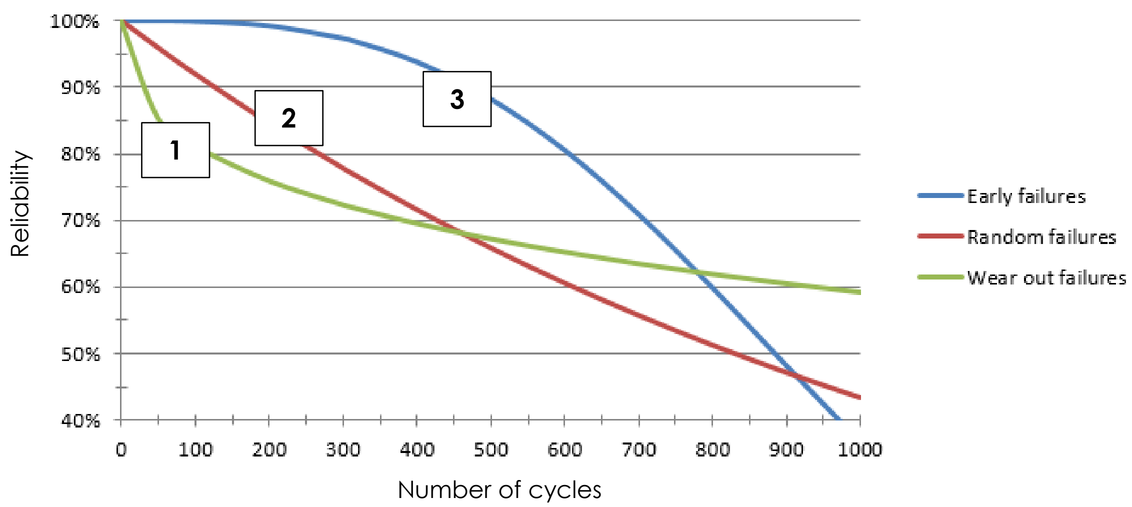


Figure 3: Reliability function for each of the 3 periods.

Early breakdown can be prevented by manufacturing monitoring plans, conformity control methods or robust engineering methods, which are not discussed in this handbook.

The objective of high mileage reliability is to push the failure modes associated with components subject to wear-out beyond the desired period (see ***Figure 4***).

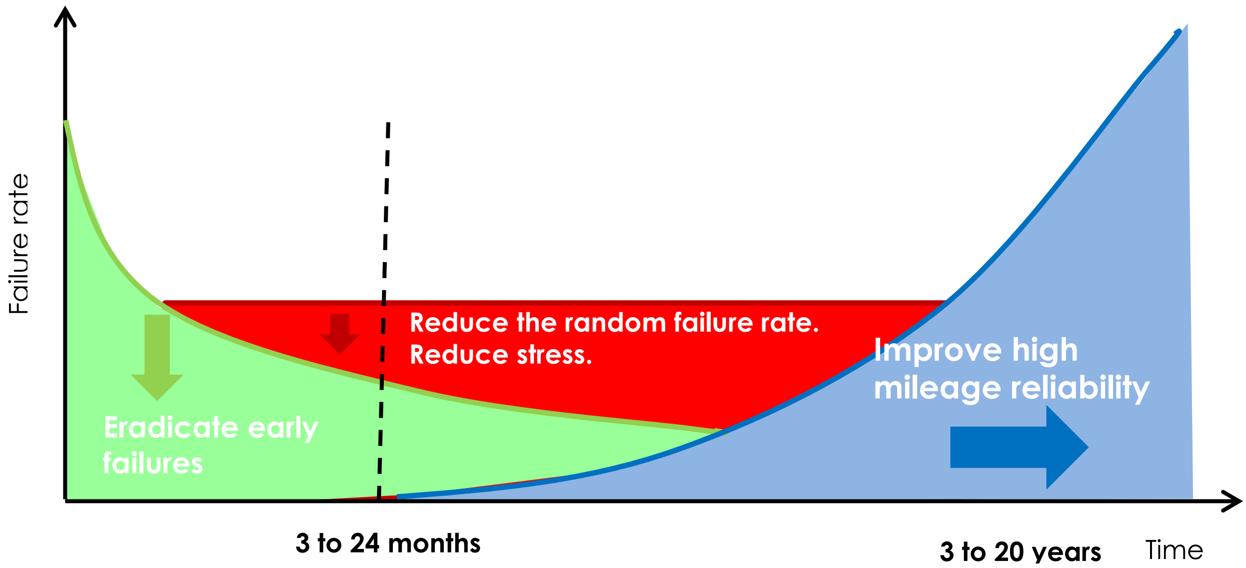


Figure 4: Issue of high mileage reliability.

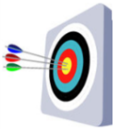
This handbook aim is providing a methodology for quantifying high mileage reliability and positioning it as necessary.

The purpose of this handbook is to present the most proven methods and not to detail all the statistical techniques. Literature references are given in **Annex 2 (themes 3 and 4)**.

This document does not develop the improvement of the design rules that can be used for designing mechanical or electrical systems. Reliability, like quality, is built on a virtuous PDCA circle. Each company improves its rules and its design standards based on this own feedback.

4 DEFINITIONS AND SCOPE

To avoid any ambiguity, it is important to discuss again the definition of the term durability and the distinction between durability and reliability (see ***Figure 5***).



Durability is the ability of an entity to perform a required function under given usage conditions and maintenance, until a limit state is reached. The limit state corresponds to the termination of the use of the entity, and can be determined by the end of life, that is to say, when the risk of failure becomes unacceptable or when the entity is considered as non-reparable after a failure.

The limit state is usually related to wear or degradation. The non-reparable state of an entity may correspond to an unacceptable repair cost. The time needed for commissioning until this limit state is called the lifetime.

In some companies, durability is associated with the concept of degradation of a performance (appearance, noisiness...) and therefore is not limited to the functional degradation.

Reliability is the ability of an entity to perform a required function under given conditions for a given time interval.

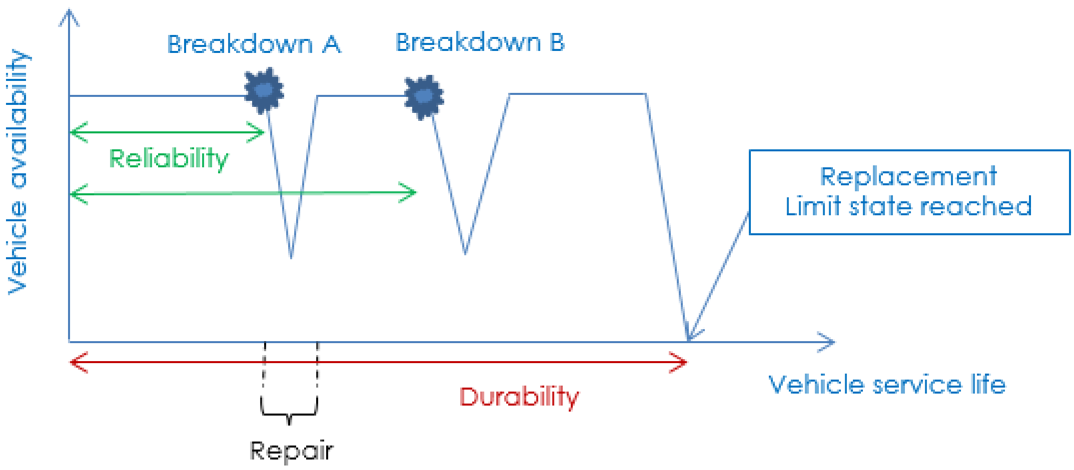


Figure 5: Reliability and durability.

This handbook focuses on the quantitative evaluation of reliability from measurements or from results of performance degradation, that is to say, used parts or damage measurements.

This quantification involves the definition and the use of a validation plan that includes numerical and physical simulations. This plan deals with electrical, electronic or mechanical failure modes. A component can fail due to various failure modes. This document focuses on component reliability. A system is considered as a structure[3] of components.

Systems and software reliabilities are not detailed in this handbook. Literature references are provided in **Annex 2 - theme 2 - [1]**.

[3] A mechatronic system consists of electrical, electronic and mechanical failure modes that have to be identified and characterized.

5 METHODOLOGY FOR DEFINING A RELIABILITY VALIDATION PLAN

The methodology proposed to quantify reliability consists of 6 phases and 2 prerequisites (***Figure 6***). It can be seen as complement to the ISO 26262 (see practical sheet 10) standard for the validation of functional safety features.

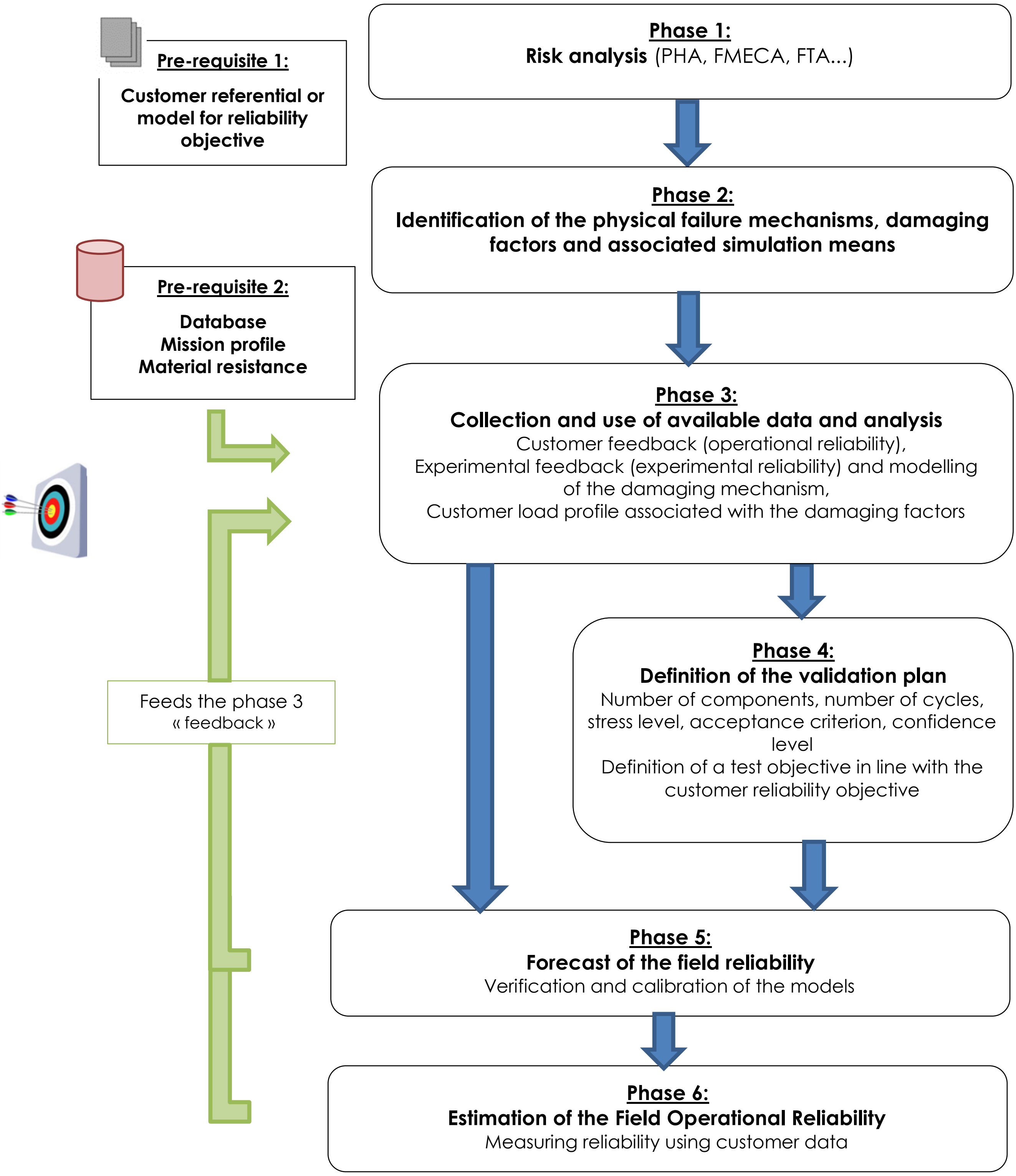


Figure 6: Methodology for defining a reliability validation plan.

Each company must define the position of its activities in its internal organization all along the projects.

However, concerning the way to effectively manage these activities, we recommend:

- To promote collaboration between specifiers, contractors and designers. This collaboration is essential to identify failure modes, relevant damaging factors and to obtain the associated input data (load profile).
- Some activities are time consuming, such as the characterization of the mission and load profiles, the definition and the application of the validation plan including the associated test means and measures. It is important to anticipate.

Table 1 lists the actors for each phase of ***Figure 6*** (with the main actor in bold type) and reports an estimation (magnitude) of the duration (in % of the total duration). This duration does not include the time needed to complete tests or numerical simulations.

Table 1: Stakeholders and order of magnitude for each phase duration (% of the total duration).

Phases	Stakeholders	Duration (total duration en %)
1. Risk analysis	**Manufacturers** Equipment manufacturers	20
2. Identification of the physical failure mechanisms, damaging factors and associated simulation means	**Equipment manufacturers** Manufacturers	20
3. Collection of available data and analysis	**Equipment manufacturers** **Manufacturers**	25*
4. Definition of the validation plan	**Equipment manufacturers** Manufacturers	25
5. Forecast of the field reliability	**Equipment manufacturers** **Manufacturers**	10
6. Estimation of the Field Operational Reliability	**Equipment manufacturers** **Manufacturers**	0 Activity carried out outside the phases of a project

* Warning: if the load profiles are not available, their characterization can take up to 80% of the time of the overall study (realization of long measurement campaigns).

The second part of the handbook deals with 2 application examples. They illustrate the various phases of ***Figure 6***. It deals with the wear of brake pads and the fatigue due to door slams. The methodology described in these examples is in line with the recommendations of the handbook.

The data used in the examples are not the actual data collected by the manufacturers but remain compatible with the industrial context.

5.1 PHASE 1: RISK ANALYSIS

Risk management is the result of an iterative process. This process consists in:

- searching and identifying the risks of a system,
- prioritizing them according to their severity and / or probability of occurrence,
- accepting or dealing with them with preventive measures (to reduce their probability) or protective measures (to reduce their severity).

Besides the assessment of margins which ensure a certain design robustness (worst case analysis, stress analysis, etc.), risks can be identified through feedback (including past failures events) or by additional methods.

The most common methods are:

- the Hazard analysis and risk assessment (HARA)
- the Failure Modes, Effects and Criticality Analysis (FMECA).

The Hazard Analysis allows identification of failure scenarios presenting a hazard to the customer, whose severity depends on the aggravating situations. The objectives of this analysis are:

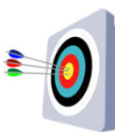
- to systematically identify all the potential Hazard Events (HE), which might endanger or compromise the missions of the system or its environment, as well as their level of severity,
- to highlight the causes and scenarios leading to these hazard events: hazard items, hazardous situations, potential accident (human error, power failure, wear, external attacks...)
- to identify risk reduction measures,
- to show that risk reduction measures are sufficient.

The analysis is performed while the system is not yet defined (black box). The functions, the environment, the implementation, the supposed technologies and especially the mission profiles are known (the input document is the "external" functional analysis).

The Failure Modes, Effects and Criticality Analysis is based on an internal functional analysis. It consists in establishing the list of potential failure modes, their causes, their effects on the system and / or the environment for each identified function or sub-function. It allows identification of the most critical risks (Risk Priority Number or RPN) and determination of the actions to implement to reduce them.

HALT tests may also be used to identify the failure modes. They are commonly used to detect weaknesses, fuses points, failure of electronic circuits subjected to vibration... **Warning**: all the failures observed during HALT tests are not necessarily representative of the defects seen by customers.

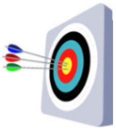
In addition, a **Fault Tree Analysis** (FTA) can be performed. (Standard NF EN 61025). This method is widely used in the analysis of reliability, availability or safety of systems. It allows one to search, from individual component failures, combinations of individual component failures that could lead to the hazard event. This logical sequence is shown graphically as a tree structure. During this phase, the FTA method is used to allocate reliability requirements by decomposition into unit reliabilities to which it is sometimes necessary to add occurrences of external events.



These analyses make it possible to highlight the main system failure modes and their associated risks. Physical failure mechanisms are indicated in the RIAC-FMD 2013 (Reliability Information Analysis Center-Failure Modes / Mechanism Distributions) for numerous types of components. They can be used as causes of failure modes.

Input data:

- Referential or model built by the company for defining the customer reliability objectives (pre-requisite 1)
- External and internal functional analyses



Output data:

- Hazard failure modes
- The hazard events and their associated severity levels. The severity level corresponds to the impact of the effect on human and material. The failure mode will be prevented using protection methods or it will be inevitable and the part will be damaged
- Potentially, the customer reliability target associated with the severity level

5.2 PHASE 2: IDENTIFICATION OF THE PHYSICAL FAILURE MECHANISMS, DAMAGING FACTORS AND ASSOCIATED SIMULATION MEANS

First, in this phase a physical mechanism of failure (degradation mechanism or physical damage likely to lead to the destruction of the material) is associated with each wear-out failure mode identified in phase 1 that cannot be solved by preventive measures.

There are multiple physical failure mechanisms. Examples would be Fatigue, or thermal fatigue, wear, corrosion, soiling or carbon build-up...

The next step of this phase consists in identifying the damaging factors at the origin of failure. They can:

- correspond directly to the stress (force, speed, time, temperature, number of operations / activations, stress, applied power, load rate, ...)
- or simply be calculated from multiple parameters (equivalent fatigue...)
- or be derived from a more complex model (see **Practical sheet 1**).

Environmental conditions (dust, humidity, sunshine...) may also increase damage. Their influence is often unclear and therefore difficult to quantify. The constraints are applied on test benches according to a profile previously defined to accelerate the emergence of failure.

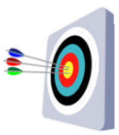
Finally, at this phase, the following items should be checked:

- with the teams responsible for "mission profiles supply," the existence of the data necessary to build the load profile of the damaging factor(s) (or component mission profile). The definition of this profile can be time consuming. It must therefore be anticipated as soon as possible.
- with the “testing" teams and / or the "computing" teams, the existence of means (numerical and / or physical) to reproduce the physical failure mechanism. If the physical means does not exist, it will be necessary to investigate the possibility to build one. Again, it is crucial to anticipate it.

- to predict with the "project" teams, the required resources as well as the planning.

Input data:

- Principal failure modes of the components
- Expertise on Serial parts
- Test results of components manufactured having the same technology and presenting the same physical failure mechanism
- Usage conditions (country, climate, road, condition…)



Output data:

- Physical failure mechanism associated with the component failure mode
- Damaging factors associated with the physical failure mechanism of the component
- Existence of data to build the load profile (mission profile of component: functional and environmental)
- Existence of a physical or numerical means for reproducing the physical failure mechanism
- Resources and planning

5.3 PHASE 3: COLLECTION OF AVAILABLE DATA AND ANALYSIS

This phase is very important because it will help to define the quantification method depending on the available feedback. It will also determine whether the load profile of the damaging factor(s) is known.

The 3 types of data described below are independent. They may be analyzed in any chronological order.

The existence of end-user feedback and experimental feedback allows the use of simple methods to design the validation plan and quantify the reliability (see **Part A.5.4**).

If end-user feedback is not available, a quantification method is recommended (Stress-Strength method). It requires to know the load profile (see **Part A.Part A.5.3.4**). The availability of experimental feedback without end-user feedback allows optimization of the validation plan (test at zero failure or bogey test) (see **Part A.5.4**).

5.3.1 END-USER FEEDBACK

The **end-user feedback** consists of in service observed failures or degradation measurements.

This activity is carried out by retrieving the estimates made during operational reliability studies in the field for a fleet of vehicles. The method is described in **Part A.5.6**.

5.3.2 EXPERIMENTAL FEEDBACK AND MODEL FOR DAMAGING MECHANISM

Experimental feedback corresponds to the statistical analysis of test results. To be relevant, the tests must be performed on components having the same technology as the studied component. They must also show the same physical failure mechanism.

This involves obtaining the parameters of the statistical law (see **Sheet 2**) defined from test data on a similar component with the same physical phenomenon leading to damage. This statistical law may correspond to a lifetime distribution used to develop a zero-failure test or a censored test, or used to be applied to an acceleration law (see **Sheet 4**) used to develop an accelerated test (e.g., the Basquin model in fatigue).

Do not use the parameters estimated from feedback if it is not sure that the physical **mechanism is the same as the one considered**. This can lead to insufficient or unrepresentative testing. Therefore, failed parts in these tests should be analyzed and the failure cause shoud le identified.

A working group made up of the companies RENAULT, PSA, ARCELOR MITTAL and CETIM built the SIA handbook entitled "Recommendations for the statistical characteristics of steel sheet fatigue strength" (DC-05). This handbook gives, for different classes of materials, orders of magnitude for the Basquin coefficient, and for the scatter of the endurance limit and the number of cycles to failure.

5.3.3 RELIABILITY HANDBOOKS

For the electronic or mechatronic components, test results have been compiled in reliability handbooks such as FIDES (UTE C80-811), MIL-HDBK-217F, MIL-HDBK-217 PLUS, or IEC 61709. These compendia are used to determine predictive reliability (see **Part A.5.5**), especially in the context of safety-critical functions subjected to the requirements of the ISO 26262 standard.

These handbooks are based on empirical or physical models calibrated from feedback data and statistical analyses. They must be used very carefully, especially when applied to new technologies.

Different reliability compendia are presented and compared, with the advantages and limitations of each, in **Practical sheet 10**.

5.3.4 LOAD PROFILE

Knowing the load profile of the damaging factor(s) is essential if no end-user feedback is available. The stress profile corresponds to the statistical distribution function in service of the factor(s) that create(s) damage.

The difficulty lies in the transformation of the vehicle mission profile to the load profile of the damaging factor(s) (also called component mission profile) over the reference period.

As seen in **Part A.5.2**, knowing this profile may imply working on a vehicle (to be planned as soon as possible) and performing mathematical modeling of measured quantities by the numerical simulation (see **Practical sheet 1**).

Caution not to confuse the mission profile of a vehicle and the load profile(s) of the damaging factor(s).It is essential to identify the load profile(s) of the damaging factor(s) **over the reference period** to quantify the reliability through testing, the mission profile not being sufficient enough.

Input data:

- Vehicle mission profile
- Parameters measured on the vehicle
- Damaging factors of the component

Output data:

- Load profile of the damaging factor(s) (also called component mission profile)

5.4 PHASE 4: DEFINITION OF THE VALIDATION PLAN

This phase consists in defining the numerical and / or physical validation plan (Test Duration, load levels, acceptance criterion, confidence level, number of components) in order to:

- demonstrate that the end user field reliability target is achieved,
- optimize the cost of an existing validation plan.

Reliability tests do not directly estimate the field failure probability. The test reliability objective is to be defined so as to be consistent with the field reliability target.

This phase defines the predictive reliability model.

The test or numerical simulation should accurately reproduce the failure mode observed in operation in the field. There is no point in designing a test causing failures that customers will never see. It is therefore important to know how representative a failure mode of an aggravated test (type HALT) is before taking any design change decision (risk of unnecessary additional cost).

Depending on the available end-user feedback and on the possibility to measure degradation, the method used to quantify reliability differs. There are 4 proven methods to estimate the field reliability (see ***Figure* 7**). The existence of end-user feedback and experimental feedback allows the use of simple methods (comparison methods described in **Part A.5.4.1**) to design the validation plan and quantify the reliability. Without customer feedback, only one method is recommended (Stress-Strength method developed in **Part A.5.4.2**). This latter requires knowledge of the load profile.

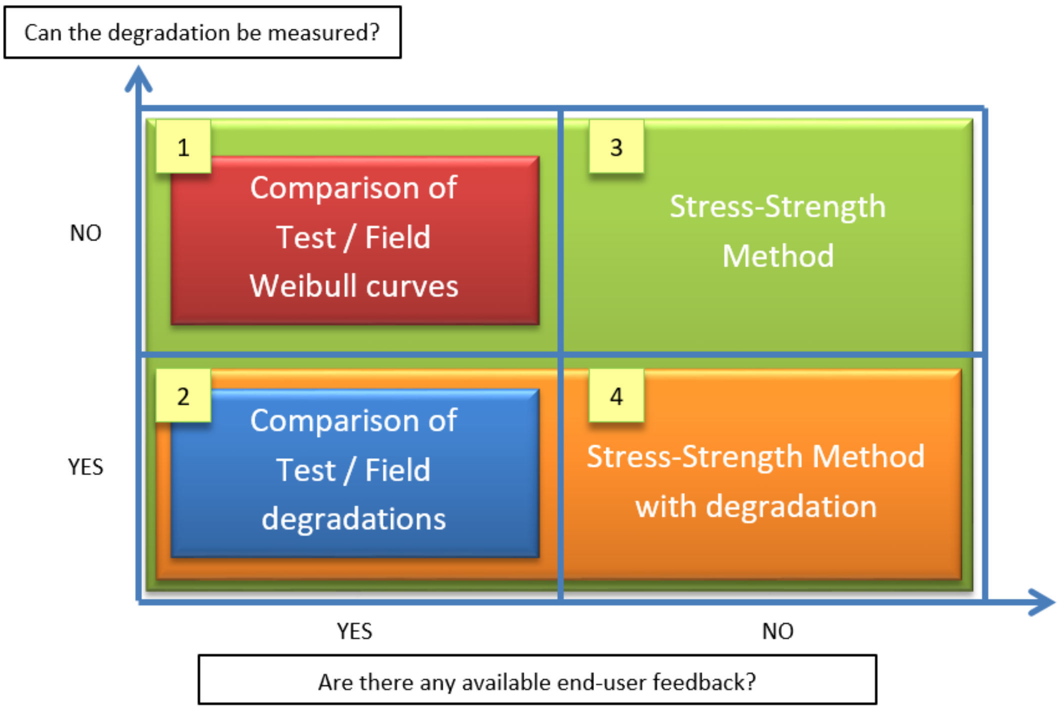


***Figure 7: The 4 methods to assess reliability*[4].**

End user feedback can be used if the components that equip sold vehicles have physical failure mechanisms similar to those studied.

[4] *Prévoir la fiabilité en clientèle à partir de données du réseau, de résultats d'essais ou de calculs. Quatre méthodes de bases illustrées par des exemples* - O. Prince, P. Schimmerling - SIA Conference « Exploitation des données du réseau pour estimer et maîtrise la fiabilité » - 12th March 2003.

The failure mode is considered as a quantifiable degradation if:

- There is a measuring or rating method to quantify the level of degradation.
- Degradation is gradual. It is then possible to extrapolate to longer periods (see **Practical sheet 8**). Corrosion and wear are typical degradations. The degradation progress, when it is linear, can be modeled with a proportional statistical distribution (linear type distribution). A non-proportional modeling (exponential, logarithmic or power law) is used to represent a non-linear progression.

5.4.1 Comparison methods

Methods for comparing Weibull distributions (Method 1) and degradation distributions (Method 2) are used to modify test acceptance criteria (or numerical simulations) based on available end-user feedback.

Thus, these methods require customer feedback.

The design of the validation plan is divided into 3 steps:

Step 1: End user field data analysis: analysis of failures (Method 1) or degradation measurements (Method 2)

End user feedback

Failed Parts with the indication of mileage and lifetime(wear) in the field

Measurements of degradation

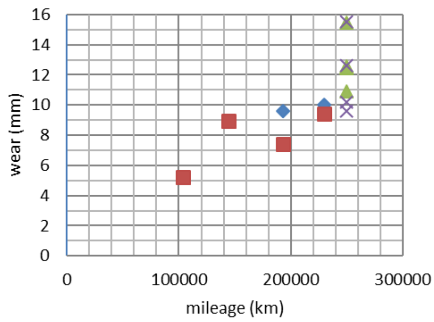


Figure 8: End user Input data.

Step 2: Analysis of the test results: failure analysis (Method 1) or degradation measurements (Method 2)

Test results

Parts that failed during test

Degradation measurements on track or during test

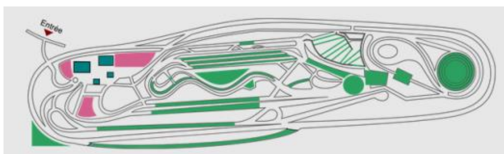


Figure 9: Input test data.

Step 3: Comparison between customer and experimental feedback and adjustment of the test (lifetime or acceptance criterion).

Figure 10 shows an example of a test adjustment. The acceptance criterion related to the test is the duration after which 10% of the components have failed (B10)[5]. This duration has been modified to be consistent with field reliability target: to achieve the target failure probability at 100 000 km instead of 50 000 km. Test duration is thus increased from 200h to 400h. Details of the method described in **Practical sheet 5**.

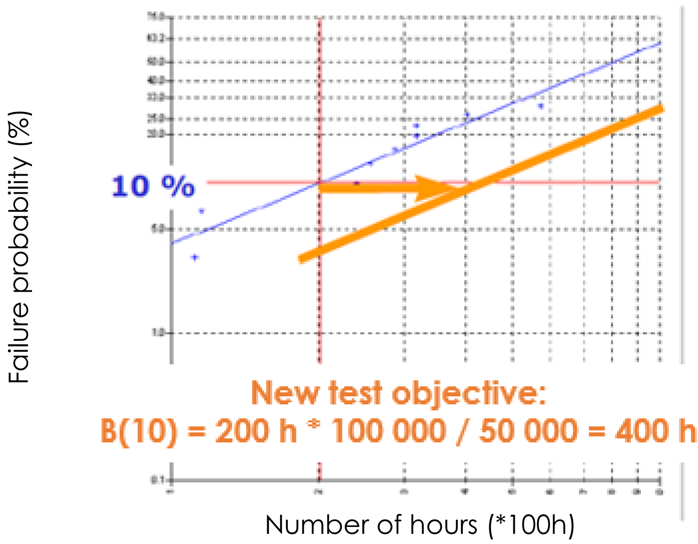


Figure 10: Adjustment of test acceptance criterion.

When comparing Test/ Field Weibull distributions, it is assumed that the test and field lifetimes are proportional.

For the method comparing the Test/ Field degradations, the mean wear observed on end-user parts is compared with the one observed during test (it is assumed that the standard deviation or the coefficient of variation of the distribution remains constant).

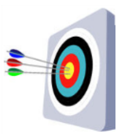
The application of these methods requires a customer technical reference which physical failure mechanism is similar to the tested reference. Similarly, field and test degradation measurements must be comparable (same type of measurement means).

Input data:

Comparison method of Test / Field Weibull curves:

- Statistical distribution (Weibull distribution) modeling the failures observed in the field
- Statistical distribution (Weibull distribution) modeling the test failures
- Field reliability estimated from field failures

Comparison method of Test / Field degradations:



- Mean degradation level and degradation variability in service
- Mean degradation level and degradation variability measured in a test
- Field reliability estimated based on degradation measurements

Output data:

- Adjusted validation plan

[5] Idem note 4 de bas de page.

5.4.2 STRESS-STRENGTH METHOD

When no end-user feedback is available, the Stress-Strength method is mainly used to quantify the reliability and define the validation plan.

Note: This method can also be used with feedback.

The Stress-Strength method is based on taking into account the variability of the applied stress C (variability of road severity, customer usage, environmental conditions...) and the variability of the component strength R (dimensional deviations, variability of material characteristics, manufacturing scatter, ...) The details of the method are described in **Practical Sheet 6**.

The method aim is to estimate the reliability of a component, taking into account simultaneously:

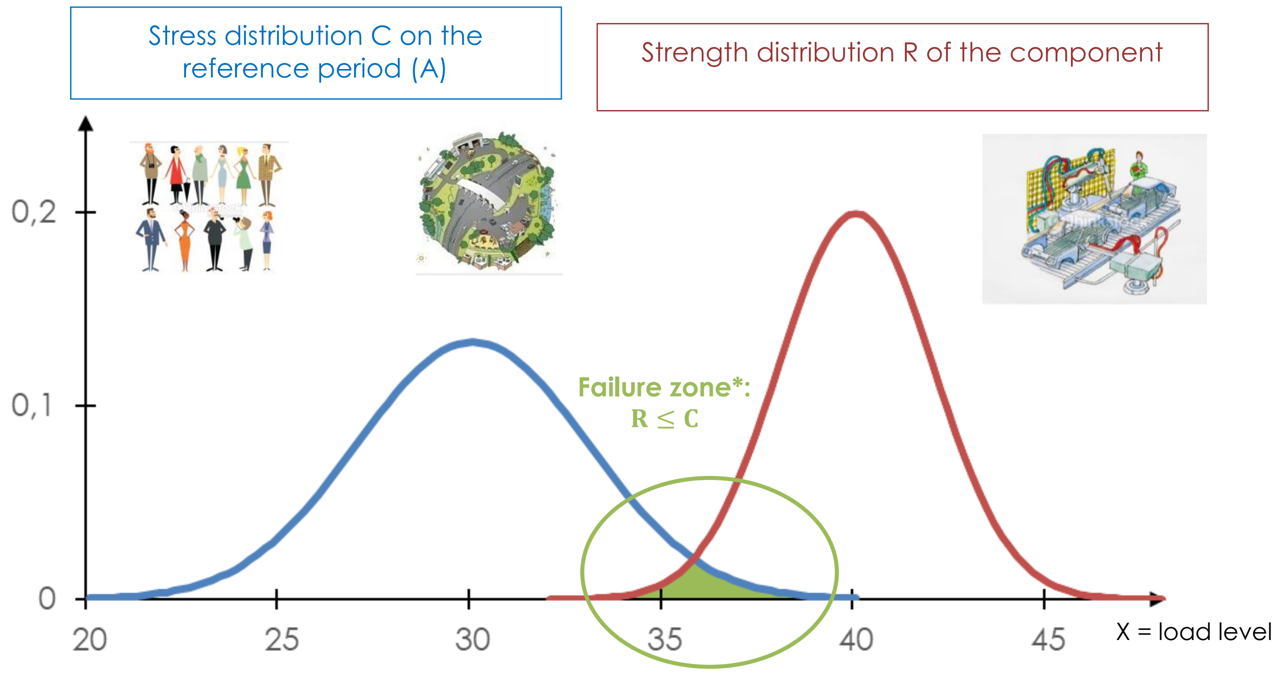
- the distribution C of the end-user loads,
- and the distribution R of components strength.

The first step consists in defining a failure condition $R \leq C$, specific to the physical damaging mechanism.

The stress C and the strength R are random variables. Indeed, components from the same production do not behave the same way (the number of cycle to failure under the same stress level cycles is different from one part to another), and each customer is not stressing their vehicle the same way (different behaviors: nervous, soft, different road types: mountain, city, highway, road...). A weak part may therefore be subjected to a severe environment.

The failure probability P_f is the probability that $R \leq C$.

The Stress-Strength method depicted in ***Figure 11*** is an approach that can be used in many cases.



*The failure probability does not correspond to the area under the curve.

Figure 11: Stress-Strength method.

It is necessary to separate the two distributions as much as possible to obtain a more reliable product. The stress distribution C cannot generally be modified by the designer because it is the result of end user loads. Thus to increase reliability (*i.e.* to reduce failure probability), the strength distribution R must be pushed as far as possible to the right of the distribution C. Another solution is to reduce the variability of the strength.

This method is simple but requires knowledge of the end-user load profile of the damaging factor(s) as well as the type of statistical distribution modeling the damage mechanism.

A variation of the Stress-Strength method consists in characterizing the strength with degradation measurements extrapolated to a degradation threshold L causing a failure. R then corresponds to the distribution of time/number of cycles to failure.

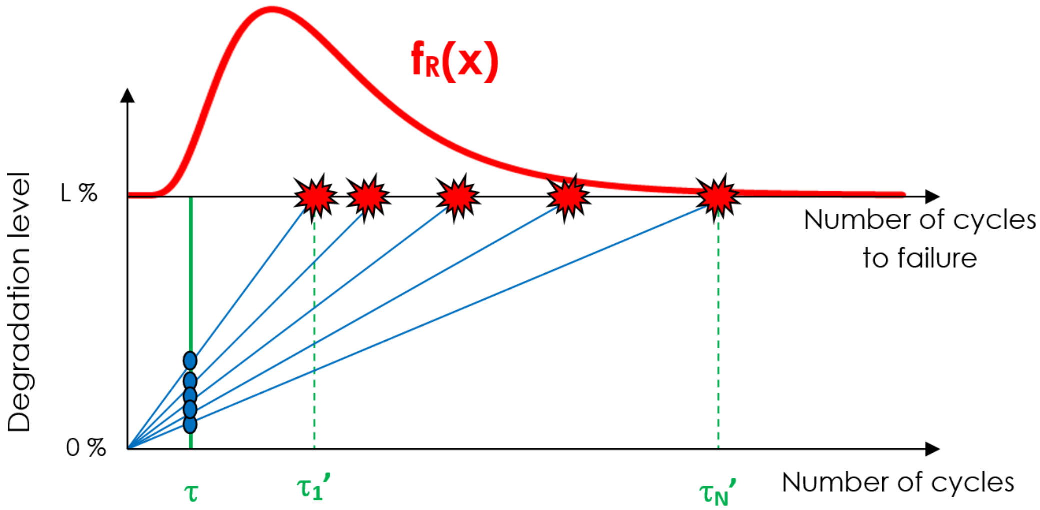


Figure 12: Extrapolation of degradation measurements observed at τ to the degradation threshold L

Input data:

- Load profile of the damaging factor(s)
- Results of field reliability
- And possibly the test of the customer component or of another component with the same technology and the same physical failure mechanism (failures observed or degradations measured on track or during test)

Output data:

- Validation plan

The demonstration of a validation plan consists in defining a test to check whether the expected reliability is sufficient. The type of test to perform depends on the available feedback.

If field and experimental feedback is available, an adjustment of the tests can be made.

If there is only experimental feedback, the current tests will be optimized (see **Practical sheet 4**, **Practical sheet 7**, **Practical sheet 8**, **Practical sheet 9**). In the absence of feedback, it is recommended to test the component to failure (destructive testing) in order to characterize the physical failure mechanism.

5.4.3 DIFFERENT TYPES OF TESTS

5.4.3.1 Zero-failure tests

The Zero-failure tests (also called time-censored tests) are tests stopped after a given period of time or number of cycles (see **Practical sheet 7, Practical sheet 7, Practical sheet 7**) set in advance.

It often consists in verifying that there is Zero failure among the N tested components after X cycles. This type of test helps to demonstrate the "experimental" reliability, that is to say, that the test failure probability is less than a given threshold.

The binomial distribution can be applied in the context of no failure tests. For a failure mechanism modeled with a Weibull distribution, the test duration can be calculated with the following formula[6]:

$$\tau = t \times \left[\frac{\ln(1-c)}{N \times \ln(1-P_f)}\right]^{\frac{1}{\beta}}$$

where:

- τ = minimum test duration (number of cycles, number of hours, mileage, etc …) for which no failure must be detected
- N = number of tested components
- t = duration for which the field reliability must be demonstrated
- $1\text{-}P_f$ = field reliability to demonstrate
- $1\text{-}c$ = accepted level of customer risk with a confidence level c
- β= shape parameter of the Weibull distribution from experimental feedback

This yields, τ = 1987 h for N = 4, t = 1000 h, $1\text{-}P_f$ = 0.95, $1\text{-}c$ = 0.2 and β = 3.

A test on 4 samples operating during 1987 hours without failure shows that the reliability $1\text{-}P_f$ is greater than 0.95 for 1000 hours of operation. When the real reliability is less than or equal to 0.95, there is 80% chance of observing at least one failure and to raise concern.

If the value of the failure probability during test δ_c $(=F_R(\tau))$ is set, it is possible to calculate the confidence level c for a given number of tested components using the following formula:

$$c = 1 - (1 - \delta_c)^N$$

Figure 13 illustrates the formula for N tested components with k= 0 failure observed. This figure shows the confidence level c in terms of the test failure probability δ_c.

It can be assumed that, with 80% confidence level, the failure probability is lower than:

- 0.149% when 10 components are tested and no failure observed
- 0.235% when 6 components are tested and no failure observed
- 0.415% when 3 components are tested and no failure observed

[6] *Reliability Demonstration in Product Validation Testing -A. Kleyner - in-Handbook of* Performability Engineering - Editor K.B. Misra - Springer -2008.

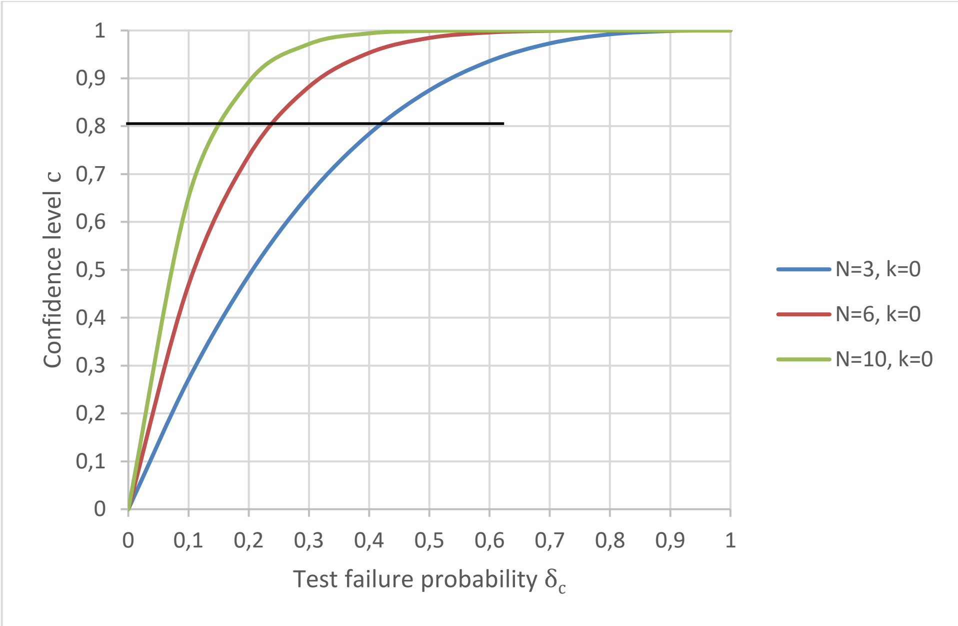


Figure 13. Confidence level in terms of the test failure probability and the number of tested components.

If a failure occurs during a test, the reliability level can also be estimated (see **Practical sheet 7** which presents the different formulas used for this type of test).

If more than 7 failures are observed before reaching the duration τ, it becomes more interesting to perform a failure analysis by adjusting a Weibull distribution (see **Practical sheet 3**). Below 7 failures, it is possible to fit a Weibull distribution provided that the parameter β is known. The β value can be set or a Bayesian interval fitting can be given (see references of the **Practical sheet 3**).

To use the feedback of an existing component (*e.g*, β of a Weibull distribution), it is necessary to prove that the new component has the same failure mode.

Remark 1:

When the failure probability is very low (close to 10^{-6}), the number of components to test becomes large. Other approaches are preferred such as accelerated tests.

The principle of an accelerated test consists in subjecting a component to higher stress than those experienced during normal use. The goal is to speed up time (duration, number of cycles...) for failures to occur earlier. The accelerated tests are representative of operational conditions (often simplified) accelerated by an acceleration factor (see **Practical sheet 4**).

An accelerated test shall not change the failure mechanisms that would be seen by customers.

Remark 2:

A level of stress set too low may induce damage which is too small to be significant and thus lead to test a large number of components. A stress level allowing the demonstration of a « B10 » is frequently selected. 10% of the components can fail at this level, with a minimum acceptable test duration ($\tau = F_R^{-1}(0.1)$).

Remark 3:

The binomial law can be used to design tests with Zero failures or with failures. However, when a part is censored before the end of the test, the binomial law does not take into account this non-failed (censored before the end of the test) part. For this, the Weibayes method can be employed (see **Practical Sheet 9**).

5.4.3.2 Complete tests

Complete tests are tests in which all the components of the sample are tested until they fail.

These tests allow characterization of the physical failure mechanism and model using a statistical law (see **Practical sheet 3**) which parameters are estimated from test data (see **Practical sheet 3**). This statistical model describes the evolution of the material failure in terms of the damaging factors. The parameters of the model characterize the strength of materials for the considered mechanism.

These analyses allow estimating one of the parameters of the probability distribution associated with the physical failure mechanism, notably the shape parameter β of a Weibull distribution or the coefficient of variation of a normal distribution or a Basquin slope in fatigue (see **Practical sheet 3**). When failure is not reached in all parts of the complete test, it is possible to determine the parameters of the statistical distribution using the MCMC method (see **Practical sheet 13**).

5.4.3.3 Censored Tests

Type 1 censored tests (also known as truncated tests) are tests stopped after a specified duration or number of cycles.

Type 2 censored tests (also known as censored tests) are tests stopped after a specified number of failed parts.

Type 3 censored tests (also known as truncated/censored tests) are tests that rely on a dual stopping condition: either after a specified duration or number of cycles OR after a specified number of failed parts.

The objective of censored tests is the same as Complete Failure Tests' but with limited test times.

5.4.3.4 Tests with degradation measurements

The principle of the method is to estimate the reliability using a validation test which will allow quantification of the probability that a degradation, quantifiable by measurement, is greater than a limit L.

In these tests (see ***Figure 12***), it is considered that the component has failed when the limit L is reached. The method is described in **Practical sheet 8**.

5.4.3.5 Numerical simulations

The result of a numerical simulation, unlike a reliability test, is not a random value. The same result is generally found when a simulation is rerun.

Most calculations do not directly quantify the probability of field failure but quantities related to reliability, *e.g.* a maximum stress level, a number of cycles to failure, from which it is possible to assess margins or safety coefficients.

However, some numerical simulation methods, such as mechanical-reliability approaches (see **Annex 2 - theme 4 - [3], [4])** and propagation of uncertainty techniques enable the estimation of the field reliability, by considering the variability of loads (usage), geometry and material strength.

5.5 Phase 5: Estimation of the Field Predictive Reliability

The first objective of this phase is to verify that the design meets the field reliability target. The second is to appreciate the risk resulting from various events, such as technical modifications, materials changes, unsatisfactory test results, new mission profile (*e.g:* extension to a new market, etc...).

The assessment of the field predicted reliability of a component can be based on the data collected in phase 3 (see **Part A.5.3**) or from the results of the validation plan of phase 4 (see **Part A.5.4**).

To assess the predictive reliability of a system, we can use methods such as those listed below:

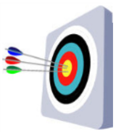
- The Fault Trees Analysis (FTA). This method is described in **Part A.5.1**. During this phase, FTA method is used to validate the overall achievement of objectives by assembling unit reliabilities (product or sum of probabilities), to which sometimes external events occurrences are added.
- The Reliability Block Diagram. This method corresponds to a graphical representation of the components of a system as well as the connections between them. It allows calculation of the reliability of a system. The diagram shows the operating status of the system based on the operating states of its components. For example, a simple series configuration indicates that all components must be functional for the system to operate. A simple parallel configuration indicates that at least one of the components must function, and so on.

Input data:

- Data collection in phase 3
- Results of the validation plan of phase 4

Output data:

- Predictive field reliability



5.6 Phase 6: Estimation of the Field Operational Reliability

The objective of this phase is to measure customer reliability based on the existence of observed failures in the field or degradation measurements in the field.
The estimation of operational reliability in customers for a component includes 3 aspects:

- identification of failures by examining the parts that failed in service,
- degradation measurements,
- statistical analysis of the failures observed in service (see Practical sheet 3).

The Weibull distribution is often used to model the failure modes.
Figure 14 presents, on the same graph, 2 failure modes of a component modeled by 2 Weibull distributions.

Data must be accurate to perform a statistical analysis: sales volume, date of entry into service, date and mileage of incident, geographic area. They should be representative of the analyzed failure mode.

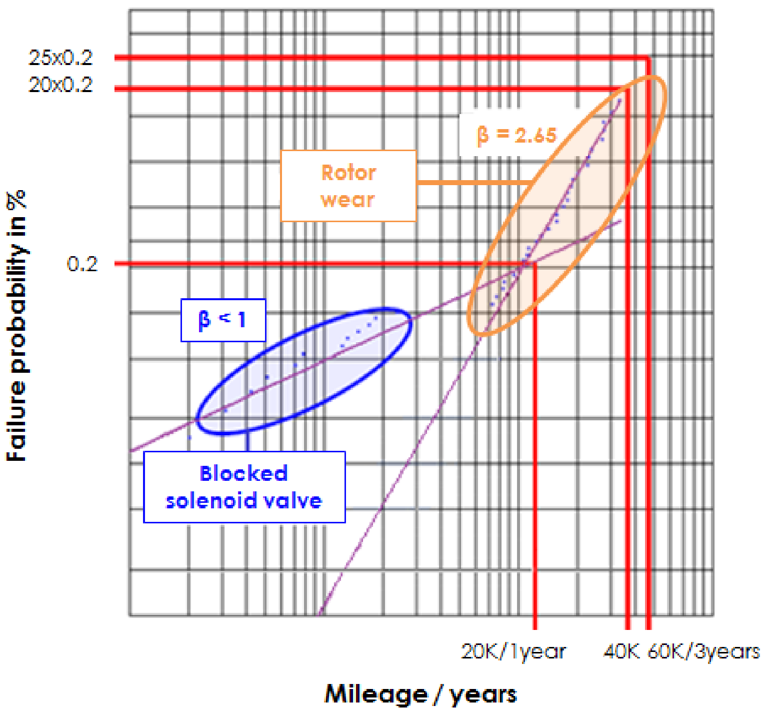


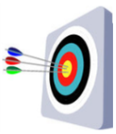
Figure 14: Modeling of the end user feedback for a variable valve timing system

Figure 14 depicts the cumulative failure probability with respect to mileage (for a variable valve timing system[7]. The distribution of failures is modeled with2 Weibull distributions. Each distribution represents a failure mode:

- the first one corresponds to an early failure caused by a manufacturing issue (β Weibull parameter <1). In the example, it is the blockage of a solenoid,
- the second one corresponds to a wear-out failure (β Weibull parameter > 1). In the example, it is the wear of the rotor.

Input data:

- Failures observed or degradations measured in service.
- Failures observed or degradations measured on a vehicle fleet.
- Vehicle statistics (number of vehicles in service, mileage distribution...)



Output data:

- Statistical distribution (*e.g* Weibull distribution) modeling field failure
- Mean level of field degradation and scatter
- Reliability estimated from failures observed and degradations measured in service

[7] *Pérenniser la Qualité en conception automobile: la démarche Design to Quality* - C. Garrel - SIA conference study day « recherche de l'efficience en qualité automobile » - 12th May 2011.

PART B. APPLICATION EXAMPLES

Ex 1. Reliability study of a brake pad

Reminder: the parameter values in this example are only indicative. They do not represent actual models and data but remain compatible with the industrial context.

1 CONTEXT/OBJECTIVE/CHALLENGE

1.1 CONTEXT

The friction of the brake pads on the disk generates wear. Below a certain thickness, braking quality is deteriorated.

1.2 OBJECTIVE

This application example aims at presenting the approach for studying the reliability of the brake pad. This approach uses the steps 1 to 4 of the reliability validation plan methodology (***Figure 6***).

1.3 CHALLENGE

The challenge is to develop an approach to verify that the field reliability target is achieved.

2 DESCRIPTION OF A DISC BRAKE

The disc brake is essentially composed of a disc integrated in the wheel hub and brake pads operated by a hydraulic mechanism, to rub them against the disc. The kinetic energy of the vehicle is transformed into heat.

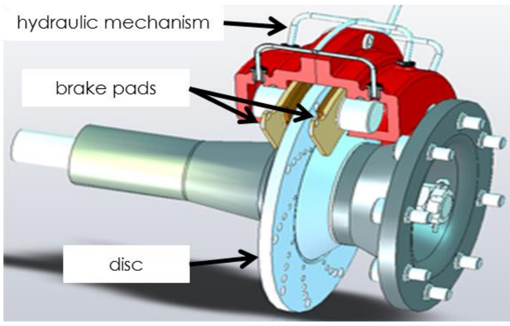


Figure 15: Disc brake[8].

[8] Reference of the figure: http://eduscol.education.fr/sti/system/files/images/ressources/techniques/2029/2029-frein-disque-dt-0.png

3 APPLICATION OF THE RELIABILITY VALIDATION PLAN METHODOLOGY

3.1 PHASE 1: RISK ANALYSIS

The Hazard Analysis (HARA) concerning the braking function of the vehicle (truck application) is given in ***Table 2***. The failure mode considered in this example is the degradation of the function. An extract of the Failure Mode Analysis, Effects and Criticality Analysis (FMECA) is given in ***Table 3***. The physical failure mechanism associated with the degraded braking function is the excessive wear of the brake pad. In the following, the field reliability objective regarding failure by abrasion is 1% at 250 000 km. The confidence level required for the design of the reliability test is 70%.

Table 2: Extract of the HARA concerning the breaking function of the vehicle.

Function: decelerate the vehicle		Phase: Driving		
Failure mode	**Scenario of appearance**	**Effect on the system and its environment**	**Undesirable customer effect**	**Severity**
No function	Moving vehicle, truck operator starts applying the service brakes. The service brakes do not work.	Vehicle does not stop/decelerate	Sudden loss of service brakes	Safety
Loss of function	Moving vehicle, truck operator starts applying the service brakes. The service brakes start to work but stop working again.	Vehicle does not stop/decelerate	Sudden loss of service brakes	Safety
Unexpected function	Moving vehicule, truck operator does not try to brake, but the service brakies start anyway.	Unintended braking	Unexpected application of service brakes	Safety
Degraded function	Moving vehicle, truck operator starts applying the service brakes. The service brakes start to work but not with full effect.	Vehicle does not stop/decelerate	Sudden loss of braking system	Safety
Misinterpreted function	No applicable			

Table 3: Extract of the FMECA indicating that a degraded braking function is due to excessive wear of the brake pad (Po: occurrence probability, S: severity, Pd: probability of detection).

Function	Failure mode	Causes of the failure mode	Effect on the system	Undesirable client effect	Means of detection	Po	S	Pd	RPN
Decelerate the vehicle	Degraded function. Braking system is not fully effective	Braking pad wear too important	The vehicle does not stop/decelerate	Sudden loss of service brakes	Test on vehicle to confirm wear model. Wear at the end of the test must be below 14.7mm	6	10	3	180

3.2 PHASE 2: IDENTIFICATION OF THE PHYSICAL FAILURE MECHANISM, DAMAGING FACTORS AND ASSOCIATED SIMULATION MEANS

3.2.1 PHYSICAL FAILURE MECHANISMS

The failure mode of the system studied is the degradation of the braking function. The physical failure mechanism studied is the wear of the pad by abrasion (phenomenon of degradation of 2 surfaces in contact and in motion). The initial thickness of the pad is 22.7 mm. The failure is defined as a pad thickness less than 8 mm, that is to say a wear of 14.7 mm.

3.2.2 DAMAGING FACTORS

In this example, the number of brake applications with sufficient pressure to damage the pads is considered as the damaging factor.

3.3 PHASE 3: COLLECTION OF AVAILABLE DATA AND ANALYSIS

3.3.1 FIELD DATA

Field wear data are gathered in the ***Table 4***.

Table 4: Field wear data.

Vehicle	Customer 1	Customer 2	Customer 3	Customer 4
Mileage	103 334	144 241	193 203	230 000
Front left wear (FLW) in mm	5.20	8.93	9.62	10.03
Front right wear (FRW) in mm	5.22	8.97	7.42	9.40

3.3.2 TEST DATA

2 accelerated tests representative of the wear phenomenon were performed. Results are reported in ***Table 5*** and ***Table 6***. Test mileage is 60 000 km.

Table 5: Data of the test 1.

	Mileage	0	4 800	14 529	23 832	34 403	44 000	54 000	60 000
Test 1	FLW (mm)	0	0.53	1.28	2.02	3.55	4.46	5.50	6.12
	FRW (mm)	0	0.56	0.98	2.02	3.20	4.17	5.46	6.27

Table 6: Data of the test 2.

	Mileage	0	6 217	11 618	20 256	30 629	41 635	53 000	60 000
Test 2	FLW (mm)	0	1.58	2.06	2.82	4.16	5.68	7.34	8.52
	FRW (mm)	0	1.62	NC	2.82	4.12	5.50	6.83	7.92

3.4 PHASE 4: DEFINITION OF THE VALIDATION PLAN

The validation plan aim is to demonstrate the achievement of the field reliability objective. 4 methods can be used to estimate field reliability through testing. The method to apply depends on the availability of field data and the possibility to measure the degradation (***Figure* 7**). In this example, the most relevant method is the comparison method of Test / Field degradations because wear is a measurable degradation and end user feedback is available. However, the other methods are also applied in this example. They require ignoring some available data, thus, generating more expensive validation plans (higher number of components or longer duration).

The **Practical sheet 15** allows to evaluate the impact of the objective parameters on the reliability validation.

3.4.1 WITH FIELD FEEDBACK

In this part the field data collected in part 3 (See **Part A.5.3**) is used for designing the validation plan.

3.4.1.1 Comparison method of Test/Field degradations (Method 2)

The comparison method for the Test/Field degradations is described in the **Practical sheet 5**. The following steps correspond to those indicated in **Part A.5.4.1**..

Step 1: End-user feedback analysis

The field wear data is extrapolated with a linear hypothesis up to the objective mileage of 250 000 km. The results are given in ***Table 7*** and illustrated in ***Figure 16***.

Table 7: Field wear data extrapolated to 250 000 km.

Vehicle	Customer 1	Customer 2	Customer 3	Customer 4
Mileage	250 000	250 000	250 000	250 000
Extrapolated FLW (mm)	12.57	15.48	12.44	10.91
Extrapolated FRW (mm)	12.62	15.54	9.60	10.22

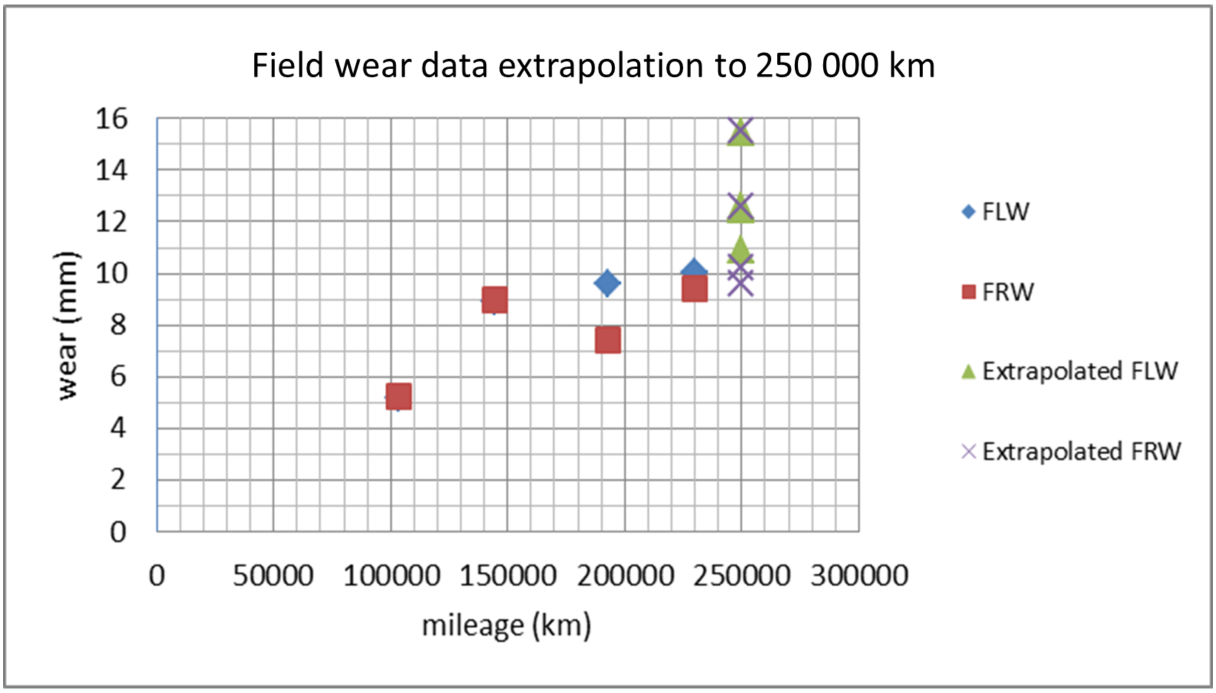


Figure 16: Field wear data extrapolation to 250 000 km.

A lognormal distribution is fitted to the wear values extrapolated to 250 000 km by a simple estimation of the parameters:

- $\mu_{\ln X}$ = 2.506 (mean wear ≈ 12.449 mm)
- $\sigma_{\ln X}$ = 0.177

The field failure probability is:

$$1 - F(14.7, \mu_{\ln X} = 2.506, \sigma_{\ln X} = 0.177) \approx 0.152$$

where F is the lognormal cumulative distribution function of the field wear at 250 000 km.

Excel ® Formula 2013: 1- LOGNORM.DIST(14.7;2.506;0.177;TRUE)

Field reliability target is not achieved 0.152 > 0.01.

Step 2: Analysis of the test results

Test wear data at 60 000 km is extrapolated linearly (hypothesis which must be validated beforehand) to the objective mileage of 250 000 km. The wear values at intermediate mileages are taken into account to build the extrapolation. The results are given in ***Table 8***.

Table 8: Test wear data extrapolation to 250 000 km.

Mileage		60 000	250 000
Test 1	FWL (mm)	6.12	25.52
	FWR (mm)	6.27	26.14
Test 2	FWL (mm)	8.52	35.49
	FWR (mm)	7.92	32.98

The mean test wear is 30.032 mm at 250 000 km.

Step 3: Comparison between field and experimental data and adjustment of the test

The test acceleration factor is estimated as the ratio of the mean wear at 250 000 km observed in test to the field mean wear at 250 000 km: 30.032/12.449 = 2.412. Therefore, 1 km in test generates on average the same wear as 2.4 km in end-user service.

As seen in step 1, the field reliability target is not met. The distribution of the end-user wear at 250 000 km is recalculated so to meet the reliability objective. The standard deviation $\sigma_{\ln X}$ is assumed constant. The new mean value $m_{\ln X}$ of the wear field distribution must satisfy:

$$1 - F(14.7, m_{\ln X}, \sigma_{\ln X} = 0.177) = 0.01$$

Excel ® Formula2013: 1 - LOGNORM.DIST(14.7;mlnX;0.177;TRUE) = 0.01

The solution of this equation is $m_{\ln X}$ of 2.277 (mean wear of 9.90 mm instead of 12.449 mm before). The new distribution is shown in ***Figure 17***.

Knowing that the mean wear customer 250 000 km is 9.90 mm and that the test acceleration coefficient is 2.412, the acceptable mean wear in test μ_T is 9.90 x 2.412 = 23.88 mm at 250 000 km or 5.73 mm at 60 000 km.

The test to be set up is designed in order to check that the mean wear μ_T is less than 5.73 mm after 60 000 km. Test can be designed using the standard ISO 39511 on sampling plans or the confidence interval formula for the mean test wear (at 60 000 km).

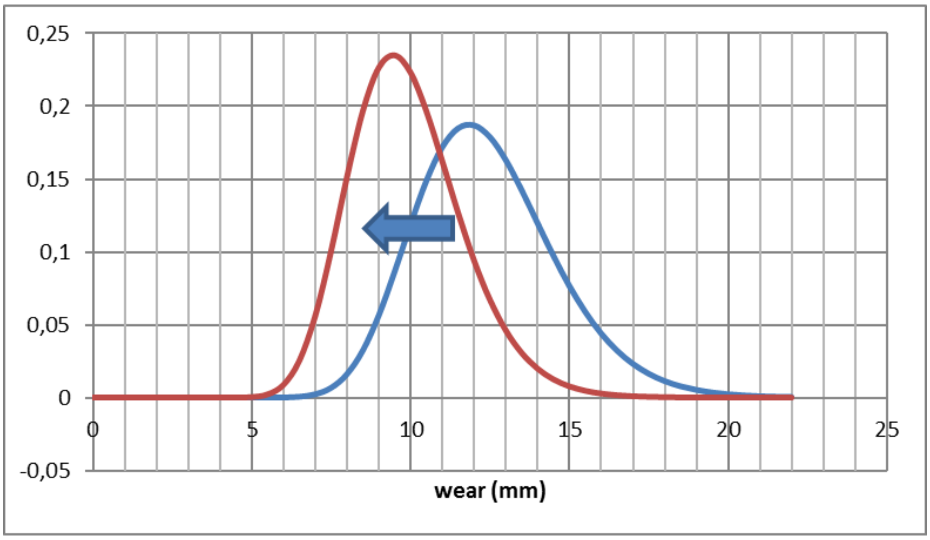


Figure 17: Adjustment of the field wear distribution for meeting the reliability target. The blue distribution becomes the red one.

10 pads are tested up to 60 000 km (***Table 9***). A lognormal distribution is assumed for the mean test wear. The mean $\overline{w}$ of ln(wear) on the sample is 1.70. The unbiased estimate s_w^* of the standard deviation of the observed wear logarithm is 0.25.

Table 9: Test example at 60 000 km for 10 pads.

# pads	Wear (mm)	ln(wear)
1	5.14	1.64
2	5.32	1.67
3	5.63	1.73
4	4.87	1.58
5	4.32	1.46
6	4.14	1.42
7	4.82	1.57
8	5.01	1.61
9	4.50	1.50
10	4.78	1.56
Mean		1.58

The mean value $\overline{w}$ is not the mean value of the population. It is necessary to calculate the confidence interval over the mean value. When a random variable is normally distributed, the upper boundary of the confidence interval of the mean value for a given confidence level c = 70 % is:

$$L_{sup} = \overline{w} + t_{c,N-1} \frac{s_w^*}{\sqrt{N}}$$

Where N is the number of pads (=10) and $t_{c,N-1}$ the quantile of the Student distribution for N degrees of freedom.

Excel ® Formula 2013: 1,58+T.INV.2T(0.7;10-1)*0.25/SQRT(10)

Now we must check that the boundary L_{sup} is below the objective value of the mean wear (in Log). For c = 70% and for 10 pads, L_{sup} is equal to 1.623 = ln (5.07), that is to say wear of 5.07 mm. So, the test mean wear has 70% chance of being below 5.07 mm. The objective of mean wear <5.73 mm is therefore achieved.

3.4.1.2 Comparison method of Test/Field Weibull curves (Method 1)

The comparison method of Test/Field Weibull curves is also applicable in this example because the data can be extrapolated to failure as it is explained in **Practical sheet 8**. Failure occurs when thickness is lower than 8 mm, that is to say a wear value of 14.7 mm. The following steps correspond to those indicated in **Part A.5.4.1**.

Step 1: Field feedback analysis

First of all, the data concerning the end-user wear are extrapolated linearly to a wear value of 14.7 mm (= failure). The results are given in ***Table 10*** and presented in ***Figure 18***.

Table 10: Field wear data extrapolated to failure.

Vehicle	Customer 1	Customer 2	Customer 3	Customer 4
Wear until failure (mm)	14.7	14.7	14.7	14.7
Mileage until failure FWL	292 305	237 352	295 329	336 977
Mileage until failure FWR	291 091	236 469	382 933	359 681

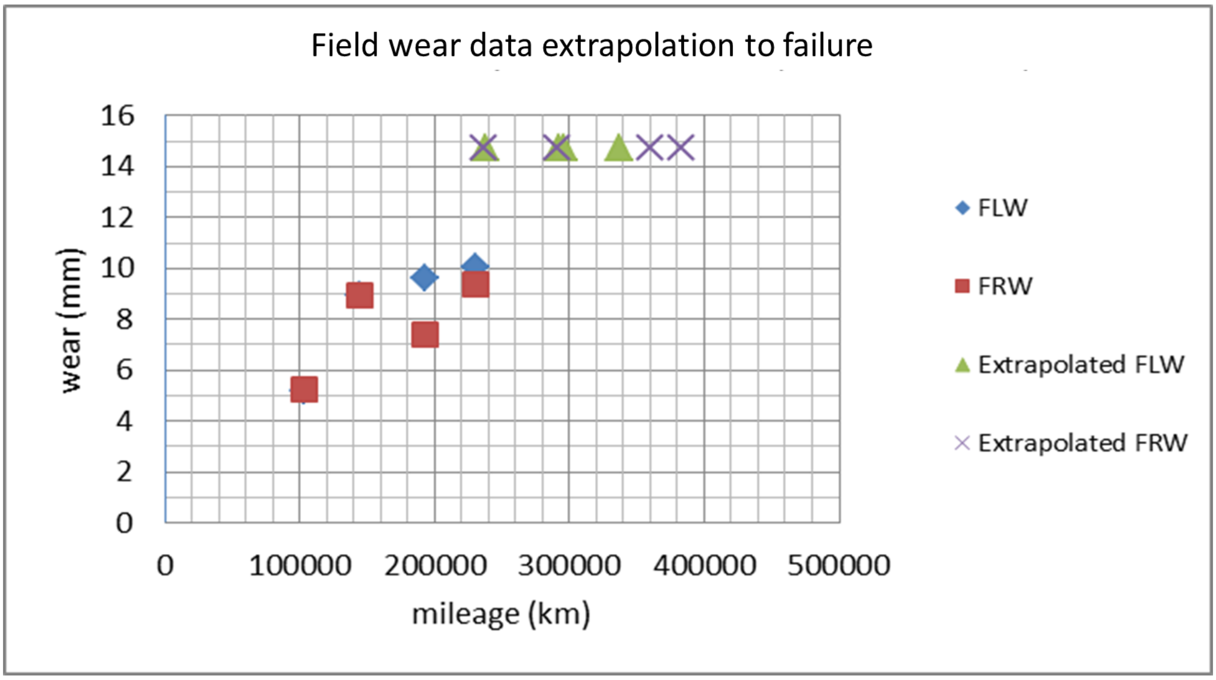


Figure 18: Field wear data extrapolation to failure.

Following this, a Weibull distribution is fitted to the field mileages to failure by the median ranks method (see references of the **Practical sheet 3**). Its parameters are β_c = 6.46 and η_c = 324 780. The reliability target is 1% at 250 000 km. The mileage value of 1% of customer pads to fail is:

$$F^{-1}(0.01, \eta_c = 324\,780, \beta_c = 6.46) = \eta_c \times [-\ln(1-0.01)]^{\frac{1}{\beta_c}} \approx 159\,342 \text{ km}$$

where F^{-1} is the inverse Weibull distribution function of the field mileage to failure.

The reliability target is not achieved because 159 342 < 250 000 km.

Step 2: Analysis of the test results

A Weibull distribution is fitted to the test mileages to failure (see ***Table 11***) using the median ranks method. The parameters of the distribution are: β_e = 6.68 and η_e = 137 493. The accelerated test mileage is 60 000 km. The failure probability before 60 000 km is:

$$G(60\,000, \eta_e = 137\,493, \beta_e = 6.68) = 0.004$$

where G is the Weibull cumulative distribution function of the test mileage to failure.

Excel formula ® 2013: WEIBULL.DIST(60000;6,68;137493;TRUE)

Table 11: Test mileage data extrapolated to 14.7 mm.

Test 1	**Failure mileage FWL (km)**	144 117
	Failure mileage FWR (km)	140 670
Test 2	**Failure mileage FWL (km)**	103 521
	Failure mileage FWR (km)	111 363

Step 3: Comparison between field and experimental feedback and adjustment of the test

Test and field failure mileage are assumed to be proportional. The new acceptance criterion is thus:

$$60\,000 \times \frac{250\,000}{159\,342} \approx 94\,137 \text{ km}$$

where 250 000 / 159 342 is the ratio between the mileage of the reliability target and the field mean failure mileage.

To achieve the field reliability target, the time-censored test (see **Practical sheet 7**) should check that no more than 0.4% of failures are observed after 94.137 km (value of the failure probability of step 2 and mileage from step 3). For a confidence level of 70%, the test should be performed on N ≈ 300 pads:

$$\mathrm{N} = \frac{\ln(1-0.7)}{\ln(1-0.004)} \approx 300$$

If no failure is detected at the end of the test, then the reliability target is achieved.

The number of pads to be tested is large, but, it is possible to reduce it by increasing test mileage. In order to do this, it is necessary to determine the adjusted Weibull distribution assuming a constant value of β_e. Knowing the point (94 137 km, 0.004) of the distribution function and β_e, the parameter η_e is determined as (***Figure 19***):

$$\mathrm{G}(94\,137, \eta_e, \beta_e = 6.68) = 0.004$$

Its value is 215 082 km.

For a confidence level of 70 % and 10 pads tested, test mileage value is:

$$\tau = \mathrm{G}^{-1}(\delta_c, \eta_e = 215\,082, \beta_e = 6.68) = \eta_e \times [-\ln(1-\delta_c)]^{\frac{1}{\beta_e}} = 156\,665 \text{ km}$$

where $\delta_c = 1 - (1-0.7)^{1/10}$ is the test failure probability for a confidence level of 70 % (see Erreur ! Source du renvoi introuvable.)

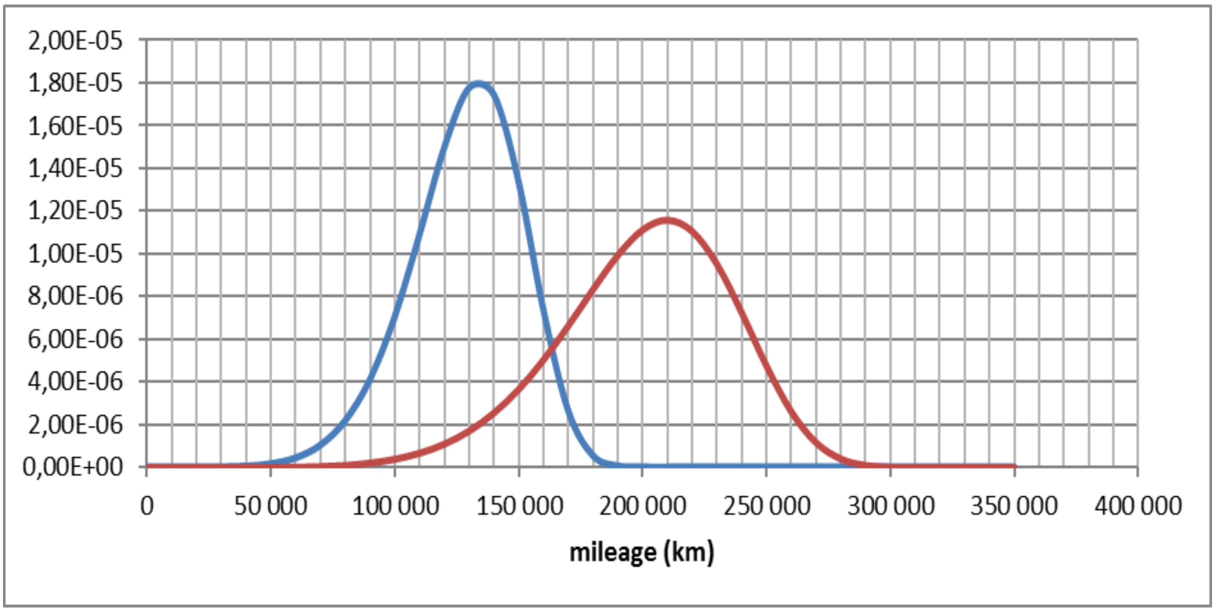


Figure 19: Test mileage increase. The probability distribution function in blue is the one from step 2. Probability distribution function in red is the new one obtained given a point on the cumulative distribution function and with a constant β_e.

The cost of the validation plan obtained with the degradation comparison Test/Field method (see **Part B.Ex 1.3.4.1.1**) is much lower (10 components, 60 000 km / vehicle) than the one obtained with the comparison method of Test/Field Weibull curves (10 components, 156 665 km / vehicle). **This shows that it is better to use the degradation comparison method when the progressive degradation is measurable. A quantitative measure brings more information than binary-type data (defective/safe).**

3.4.2 WITHOUT FIELD DATA FEEDBACK

Here, it is assumed that no field data (see **Part A.5.3**) is available. The design of the validation plan is thus carried out using the Stress-Strength method (Methods 3 or 4).

Step 1: Determination of the failure condition

As indicated in **Part B.Ex 1.3.2.2**, the damaging factor considered in this example is: the number of brake applications with sufficient pressure to damage the pads. The failure condition is defined following this factor.

Step 2: Definition of the stress distribution

The following stress distribution is provided by the team in charge of building the mission profiles. A lognormal distribution with an expected value $E[C]$ = 22 000 and a standard deviation σ_C = 18 000 (thus $\mu_{\ln C} \approx 9.74$ and $\sigma_{\ln C} \approx 0.72$) is used to describe the number of damaging brake applications on the reference period of 250 000 km.

Step 3: Determination of the strength distribution which meets the reliability target

The purpose of the Stress-Strength method is to determine the parameters of the strength distribution that meets the reliability target. A lognormal distribution is assumed for the strength.

The following formula taken from the **Practical sheet 6** allows us to express the parameters $\mu_{\ln R}$ and $\sigma_{\ln R}$ of the strength distribution in terms of the reliability objective and the parameters $\mu_{\ln C}$ and $\sigma_{\ln C}$ of the stress distribution:

$$P_f \geq \Phi\left(-\frac{\mu_{\ln R} - \mu_{\ln C}}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln C}^2}}\right)$$

Where P_f is the failure probability target (=in the field) and Φ is the standard cumulative distribution function (normal distribution) with a mean value 0 and a standard deviation value 1).

Excel ® formula2013:

mu_ln_R = -NORM.S.INV(Pf)*SQRT(Sigma_ln_R^2+Sigma_ln_C^2)+mu_ln_C

A design graph is constructed from this formula. It is given in ***Figure 20***. Only the solutions located in the OK zone (under the curve) allow the verification of the field reliability target.

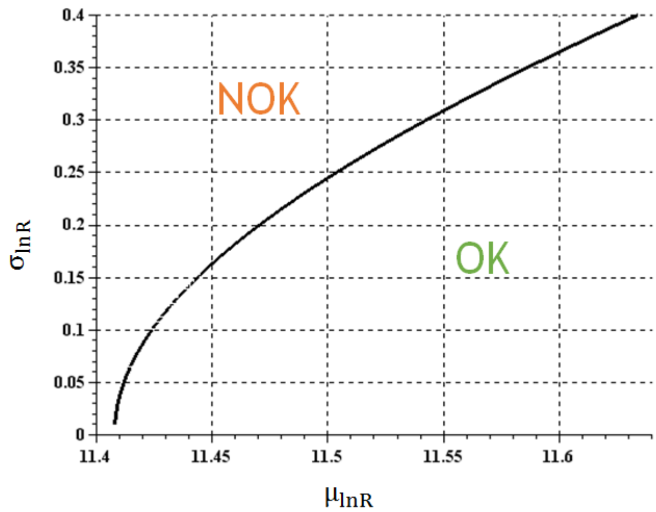


Figure 20: Design graph of couples ($\mu_{\ln R}$; $\sigma_{\ln R}$) used to achieve the field reliability target.

Step 4: Test design

With experimental feedback

Experimental feedback allows to define one of the 2 strength parameters (generally the coefficient of variation, standard deviation or the shape parameter β for the Weibull distribution). Time-censored tests (see **Practical sheet 7**) are sufficient to estimate the second parameter.

Assuming that the strength standard deviation $\sigma_{\ln R}$ is 0.15, the strength mean value $\mu_{\ln R}$ is then approximately 11.44 according to the design graph of ***Figure 20***.

Test duration is 60 000 km. For a confidence level of 70%, the number of pads to be tested is:

$$N = \frac{\ln(1-0.7)}{\ln\left(1 - F_R(60\,000, \sigma_{\ln R} = 0.15, \mu_{\ln R} = 11.44)\right)} \approx 686$$

Excel ® formula 2013: LN(1-0,7)/LN(1-LOGNORM.DIST(60000;11,44;0,15;TRUE))

This accelerated test requires a much larger number of pads to be tested than the comparison method of Test/Field degradations for the same mileage.

If no pad has failed at the end of the test, the field reliability target is achieved with a confidence level of 70%. Otherwise, it is necessary to test new pads or to decrease the confidence level (see **Practical sheet 7**).

It should be noted that the number of pads to be tested can be reduced by increasing the test mileage. For example, a 80 000-km test on 7 pads is enough for a confidence level of 70%.

Without experimental feedback

If no experimental feedback is available, a failure test must be performed to characterize the physical failure mechanism. As indicated in **Practical sheet 3**, at least 7 failures are required to fit a statistical distribution. The 2 estimated parameters of this distribution are reported on the design graph defined in step 3 to check the compliance with the customer reliability target.

In this example, the deterioration phenomenon is measurable. The failure test can then be advantageously replaced with a time-censored test extrapolated to failure (see **Practical sheet 8**). This is called the Stress-Strength method with degradation (Method 4).

Ex 2. Reliability study of a door

Reminder: the parameter values given in this example are only indicative. They do not represent actual distribution and data but remain compatible with the industrial context.

1 CONTEXT/OBJECTIVE/CHALLENGE

1.1 CONTEXT

A door slam generates significant stresses in the door components. The repetition of this event may result in the initiation of cracks.

1.2 OBJECTIVE

The aim of this example is to present the approach used for the reliability study for a door slam (structure and equipment). This approach illustrates the steps 1 to 4 of the reliability validation plan methodology (***Figure 6***).

1.3 CHALLENGE

The challenge is to develop an approach to verify that the field reliability target is achieved.

2 DESCRIPTION OF A DOOR

A door is composed of a metallic structure and of equipment (sees ***Figure 21***).

- Hinges
- Door stop
- Casing
- Etc…
- Door panel
- Rearview mirror
- Window-lift
- Static and dynamic window seals
- Lock
- Etc…

About 80 components are potentially concerned.

Figure 21: Equipment of a door.

3 APPLICATION OF THE RELIABILITY VALIDATION PLAN METHODOLOGY

3.1 PHASE 1: RISK ANALYSIS

The hazard analysis is given in ***Table 12***. The physical failure mechanism studied in this example is the mechanical fatigue. The field reliability objective of the system is 1 % of failure on the reference period A of 250 000 km or 15 years (whichever comes first). This objective covers the entire door system: the metal structure (housing ...) and equipment (door panel, window lift...). It is considered as an unacceptable failure probability.

In this example, a confidence level of 90% is chosen to design the reliability test in order to surely detect if the goal is not achieved. Indeed, the test will correctly detect that the level of reliability is not sufficient in 9 out of 10 times.

Table 12: hazard analysis.

Client effect	System failure mode	Physical failure mechanism	External damaging factors	Validations
Some equipment is not operating, noise…	Crack on the practical sheet metal of the door / interface window lift…	Fatigue of the components	- Slam speed - Number of slams	Endurance slam test

3.2 PHASE 2: IDENTIFICATION OF THE PHYSICAL FAILURE MECHANISM, DAMAGING FACTORS AND ASSOCIATED SIMULATION MEANS

3.2.1 PHYSICAL FAILURE MECHANISMS

The failure mode of the system studied is the initiation of cracks in the structure. The physical failure mechanism considered is mechanical fatigue. Fatigue is a process (series of mechanisms) that, under the action of stresses or strains varying with time, modifies the local properties of the materials. It may lead to the initiation of cracks and, potentially, to the rupture of the structure.

Other failures such as wear are possible but are not considered in this example.

3.2.2 DAMAGING FACTORS

The damaging factors considered in the example are the number of slams and their speed (the force depends directly on the speed).

3.3 PHASE 3: COLLECTION OF AVAILABLE DATA AND ANALYSIS

The following information is provided by the team responsible for establishing the mission profiles.

- A distribution describing the variability of the end-user slam speed: log-normal distribution with a mean $\mu_{\ln V}$ = 0.25 and a standard deviation $\sigma_{\ln V}$ = 0.11, which corresponds to an expected value of 1.29 m / s and a standard deviation of 0.142 m/s (see **Practical sheet 2**).
- A distribution describing the variability of the number of slams observed in service over the customer reference period (15 years or 250 000 km): Weibull distribution with parameters β = 1.2 and $\eta = 3 \times 10^4$.

The slams number and speed are assumed to be 2 independent random variables.

The calculation-test team provides the Basquin acceleration model (see **Practical sheet 4**) linking the slam speed to the number of slams to failure:

$$\mathrm{N\,V^b = B}$$

where:

- N is the number of slams to failure,
- V is the speed of slams,
- b = 6 is the Basquin coefficient (inverse of the straight line slope in ***Figure 22***, see **Practical sheet 4**),
- and B is a constant whose value is not important here.

3.4 PHASE 4: DEFINITION OF THE VALIDATION PLAN

The aim of the validation plan is to demonstrate the achievement of reliability targets. Its design can be used to estimate the number of test cycles and the number of doors to be tested. 4 methods can be used to estimate customer reliability through testing. The method to apply depends on the availability of field data and the possibility to measure the degradation (***Figure 7***). In this example, the most relevant method is the Stress-Strength method (Method 3) Indeed, the degradation is not measurable and no end user feedback regarding failures is available.

3.4.1 STEP 1: DETERMINATION OF THE FAILURE CONDITION

In this example, 2 damaging factors are considered: the number of slams and the speed of slams. The failure condition can be expressed either in terms of the number of slams at a given speed or by an equivalent speed for a given number of slams (***Figure 22***). The acceleration model is used to build one of these 2 synthetic parameters. Assuming that a failure test of n slams at a speed v is conducted, it is possible to determine the equivalent number of slams $\mathrm{n_{eq}}$ at a reference speed $\mathrm{v0}$. By construction, $\mathrm{n_{eq}}$ slams at $\mathrm{v0}$ during a test produce the same damage as n customer slams at a speed v:

$$\mathrm{n_{eq} = n\left(\frac{v}{v0}\right)^b}$$

This is indicated by the arrow 1 in ***Figure 22***. In this case, the failure condition is that the equivalent number of end-user slams, at $\mathrm{v0}$, exceeds the equivalent number of slams at $\mathrm{v0}$ of the failure test.

Similarly, it is possible to determine the equivalent speed $\mathrm{v_{eq}}$ for a reference number of slams $\mathrm{n0}$. By construction, $\mathrm{n0}$ cycles at the equivalent speed $\mathrm{v_{eq}}$ during test produce the same damage as n end-user slams at the speed v:

$$\mathrm{v_{eq} = v\left(\frac{n}{n0}\right)^{1/b}}$$

This operation is indicated by the arrow 2 of the ***Figure 22***. In this case, the failure condition is that the equivalent speed of $\mathrm{n0}$ customer slams is greater than the equivalent speed of $\mathrm{n0}$ failure test slams.

In the following the Stress-Strength method is applied with these 2 failure conditions. The reference values are $v0$ = 1.6 m/s and $n0$ = 80 000.

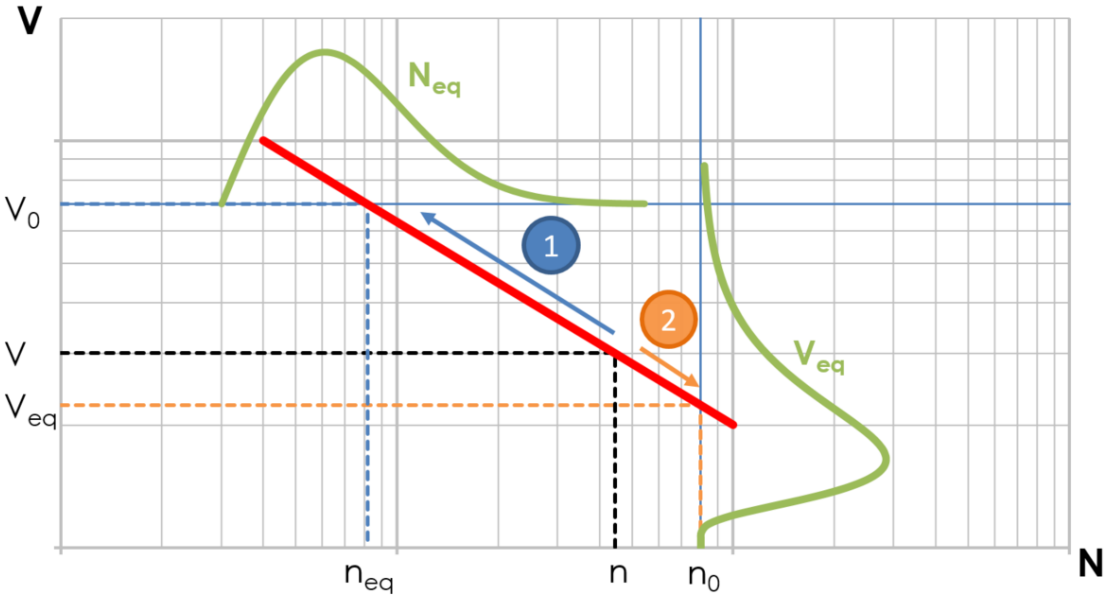


Figure 22: Acceleration model and representation of the strength distribution in equivalent number of slams at a given speed (N_{eq}) or in equivalent speed for a given number of slams(V_{eq}).

3.4.2 STEP 2: DEFINITION OF THE STRESS DISTRIBUTION

The stress distribution can either be the distribution of the equivalent number of end user slams at a given speed, or the distribution of the equivalent end user slam speed for a given number of slams. It has no explicit expression: its model distribution and its parameters are not known. This is because it is the combination of 2 distributions modeling the variabilities of the end-user slam speed and the number of slams on the customer reference period.

The stress distribution is built from:

- a Monte Carlo method (see **Annex 2 - theme 4 - [3], [4]**): random sampling in the distribution of the number of end-user slams and end-user slams speed,
- and the acceleration model that can calculate the n_{eq} or v_{eq} value for each sample of the Monte Carlo method.

The stress distribution can be described by fitting a distribution on the values of n_{eq} or v_{eq} obtained by Monte Carlo. This distribution is then used directly in step 3. An example is given in ***Figure 23***.

However, while this simplifies the next steps of the approach it introduces an additional level of approximation which can be a problem if the failure probability target is very low (distribution tails are poorly approximated). It is therefore better to keep the original random variables and to perform a random sampling in these variables to finally assess how many times the stress exceeds the strength. The estimated failure probability is the number of times where the stress exceeds the strength on the size of the sampling. This is the chosen approach for this example.

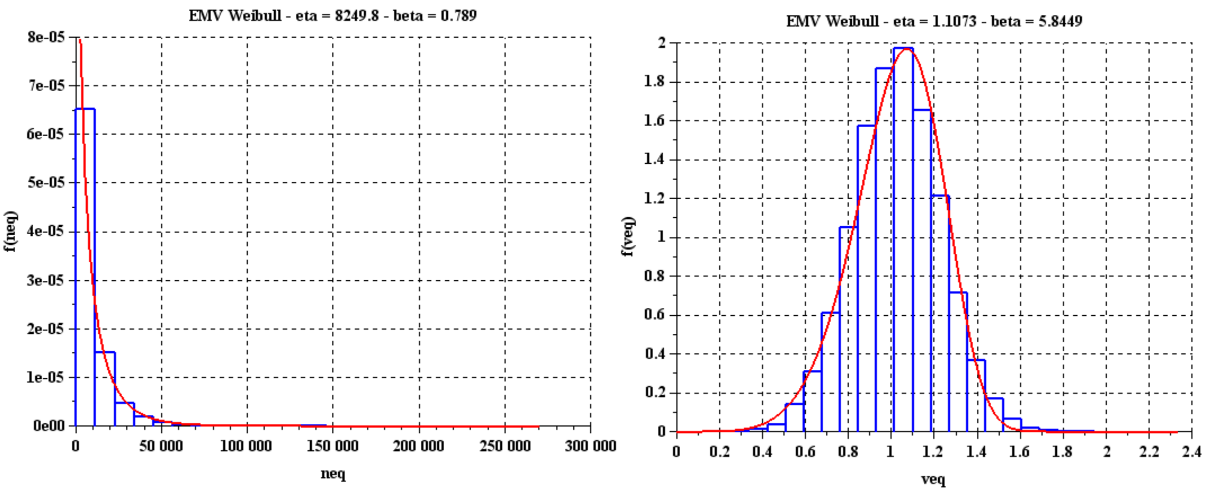


Figure 23: Stress distribution example, N_{eq} (on the left) and V_{eq} (on the right). A Weibull model is used to fit each distribution by maximum likelihood estimation about a sampling of 50 000 simulated values.

3.4.3 STEP 3: DETERMINATION OF THE STRENGTH DISTRIBUTIONS WHICH MEET THE RELIABILITY TARGET

The purpose of the Stress-Strength method is to determine the parameters ($\theta_{R,1}$; $\theta_{R,2}$) of the strength distribution which allow the reliability target to be met. According to **Practical sheet 6**, the solutions for ($\theta_{R,1}$; $\theta_{R,2}$) are obtained with the following equation:

$$P_f = \text{Prob}(R \leq C) = \int_{-\infty}^{+\infty} F_R(x, \theta_{R,1}, \theta_{R,2}) . f_C(x, A) dx$$

where:

- P_f is the field target probability of failure (= field reliability target = 1 % over the reference period A),
- R is the strength,
- C is the stress,
- x is either the variable n_{eq} or v_{eq},
- $F_R(x, \theta_{R,1}, \theta_{R,2})$ is the strength cumulative distribution function,
- and $f_C(x, A)$ is the probability density function of the stress variable.

The solutions are given as a design graph built based on Monte Carlo method. They are presented in the following for both variables n_{eq} and v_{eq}.

Remark: Design graphs are built using statistical tools depending on the company.

3.4.3.1 Design graph of the strength distribution expressed as the equivalent number of slams at a given speed (variable n_{eq})

The design graph of the strength distribution parameters (log-normal law) in terms of the equivalent number of slams at a given speed is presented in ***Figure 24***. Only the couples (μ_{lnR} ; σ_{lnR}) located in the OK zone (under the curve) are compliant with the field reliability target.

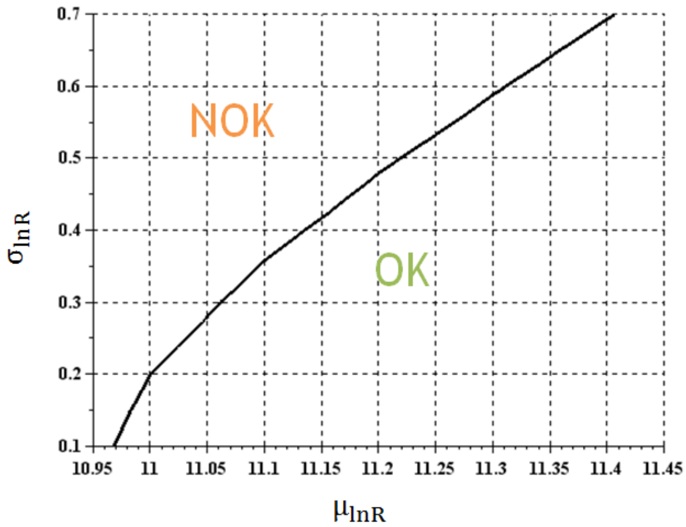


Figure 24: Design graph of couples ($\mu_{\ln R}$; $\sigma_{\ln R}$) used to achieve the field reliability. The variable is n_{eq}.

3.4.3.2 Design graph of the strength distribution expressed as the equivalent slam speed for a given number of slams (variable v_{eq})

The design graph of the strength distribution parameters in terms of the equivalent slam speed at a given number of slams is given in ***Figure 25***. Only the couples (μ_R; CV_R) located in the OK zone (under the curve) are compliant with the reliability target.

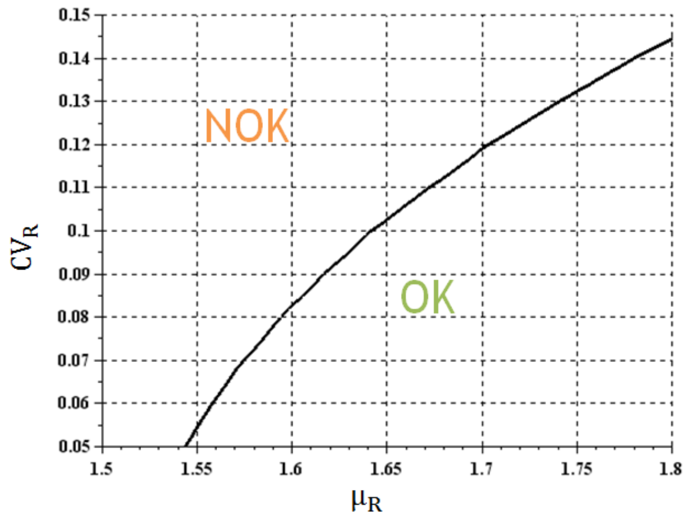


Figure 25: Design graph of couples (μ_R; CV_R) used to achieve field reliability target.

3.4.4 STEP 4: TEST DESIGN WITH EXPERIMENTAL FEEDBACK

If the experimental feedback allows estimation of one of the 2 parameters of the strength distribution, time-censored tests (see **Practical sheet 7**) are sufficient to estimate the second parameter. In this example, the experimental feedback corresponds to the coefficient of variation of the strength variable (n_{eq} or v_{eq}).

3.4.4.1 Test design – variable: equivalent number of slams at a given speed (n_{eq})

A lognormal distribution is assumed for the strength variable defined in n_{eq} (distribution N_{eq} in ***Figure 22***). The feedback allows to estimate the value of the coefficient of variation CV_R=0.5 (so $\sigma_{lnR} \approx 0.47$). Given this parameter and the Basquin coefficient (b = 6), the mean value of the strength variable μ_{lnR} must be at least 11.22 according to the design graph of ***Figure 24*** to be compliant with the field reliability target.

The test speed is $v0$ = 1.6 m/s and N = 2 doors are tested. For a confidence level of 90 %, the number of cycles Ne of the test is:

$$Ne = F_R^{-1}\left(1-(1-0.90)^{1/N}, \mu_{\ln R} \approx 11.22, \sigma_{\ln R} \approx 0.47\right) \approx 93\,520$$

Excel ® formula2013: LOGNORM.INV(1-(1-0.9)^(1/2);11.22;0.47)

If no door has failed at the end of the test, then the field reliability target is achieved with a confidence level of 90 %. If not, it is necessary to test additional doors or to reduce the confidence level (see **Practical sheet 7**). Other possible test plans are given in ***Table 13***.

Table 13: Examples of test plans.

# test plan	Number of doors tested	Confidence level	Number of slams Ne
1	1	50 %	74 608
2	1	75 %	102 438
3	1	90 %	136 261
4	2	50 %	57 750
5	2	75 %	74 608
6	2	90 %	93 413
7	3	50 %	50 763
8	3	75 %	63 836
9	3	90 %	77 830

3.4.4.2 Test design – variable: equivalent slam speed for a given number of slams (v_{eq})

A normal distribution is assumed for the strength variable defined with v_{eq} (distribution V_{eq} in ***Figure 22***)[9]. The value of the coefficient of variation CV_R is 8.8 % according to feedback. Given this parameter and the Basquin coefficient (b = 6), the mean value μ_R of the strength variable must be at least 1.61 m/s according to the design graph of ***Figure 25*** to comply with the field reliability objective.

The test duration is $n0$ = 80 000 slams and N = 2 doors are tested. For a confidence level of 90 %, the speed of slams during test Ve is:

$$Ve = F_R^{-1}\left(1-(1-0.90)^{1/N}, \mu_R = 1.61, CV_R = 0.088\right) \approx 1.68\ \mathrm{m/s}$$

Excel ® formula 2013: NORM.INV(1- (1-0.9)^(1/2);1.61;1.61*0.088)

If no door has failed at the end of the test, the customer reliability objective is achieved with a confidence level of 90 %. If not, then it is necessary to test additional doors or to reduce the confidence level (see **Practical sheet 7**).

---

[9] If a lognormal distribution is assumed for N_{eq}, the equivalent speed V_{eq} for $n0$ also follows a lognormal distribution due to the linearity in logarithmic scale of the acceleration model. In this example, a normal distribution is assumed for the equivalent speed as often recommended in literature. The results of the approaches with N_{eq} and V_{eq} will therefore be different in this example.

3.4.5 STEP 4 BIS: RUNNING A FAILURE TEST

If no feedback data is available, failure tests must be performed. It allows characterization of the physical failure mechanism and determination of the Basquin coefficient b if the door is tested at different speeds.

Table 14 shows a failure test conducted on 12 doors as recommended by the standards (**Annex 2 - theme 6 - [2],[3]**). The least squares method is applied on the number of slams N_{moy} per speed V to determine the acceleration model as (***Figure 26***):

$$\log N = \log B - b \log V$$

The value of the coefficient b is 6.942. The coefficient b is determined based on the failures of any door equipment. It is also possible to fit the failure data with only some specific components.

Table 14: Results of the failure test performed on 12 doors.

Number of the tested door	Speed V (m/s)	Number of slams N	Weibull parameters	Average number of slams $N_{moy} = F^{-1}(50\%)$
#1	1.65	100 000	β = 2.389 η = 99 971	85 751
#2	1.65	80 000		
#3	1.65	66 000		
#4	1.65	85 000		
#5	1.9	42 000	β = 2.345 η = 31 272	26 747
#6	1.9	28 000		
#7	1.9	24 000		
#8	1.9	15 000		
#9	2.2	20 000	β = 1.703 η = 14 400	11 612
#10	2.2	14 000		
#11	2.2	10 000		
#12	2.2	5 000		

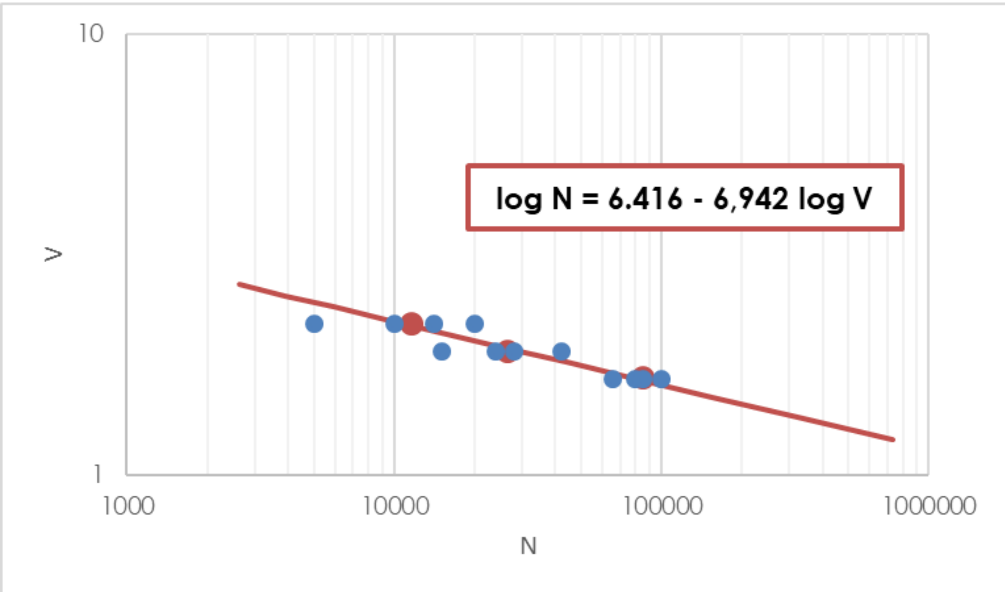


Figure 26: Linear regression estimate of the fatigue slope in log-log scale. The slope is 6.942.

3.4.5.1 Evaluation of the reliability based on the test results - variable: equivalent number of slams at a given speed (= n_{eq})

For each door, the equivalent number of slams n_{eq} at $v0 = 1.6$ m/s is calculated using the acceleration model. The values n_{eq} are reported in ***Table 15***. The median ranks method (see references of **Practical sheet 3**) is applied to fit a lognormal distribution to n_{eq} values. The results are given in ***Table 16*** and presented in ***Figure 27***.

Table 15: Equivalent number of slams at v0 for the 12 doors tested.

Number of the door	Equivalent number of slams n_{eq} at v0
#1	123 815
#2	99 052
#3	81 718
#4	105 243
#5	138 470
#6	92 313
#7	79 126
#8	49 454
#9	182 443
#10	127 710
#11	91 222
#12	45 611

Table 16: Strength distribution parameters obtained with the median ranks method.

Confidence level c	10 %	50 %	90 %
$\mu_{\ln R}$	11.682	11.457	11.227
$\sigma_{\ln R}$	0.424	0.442	0.432

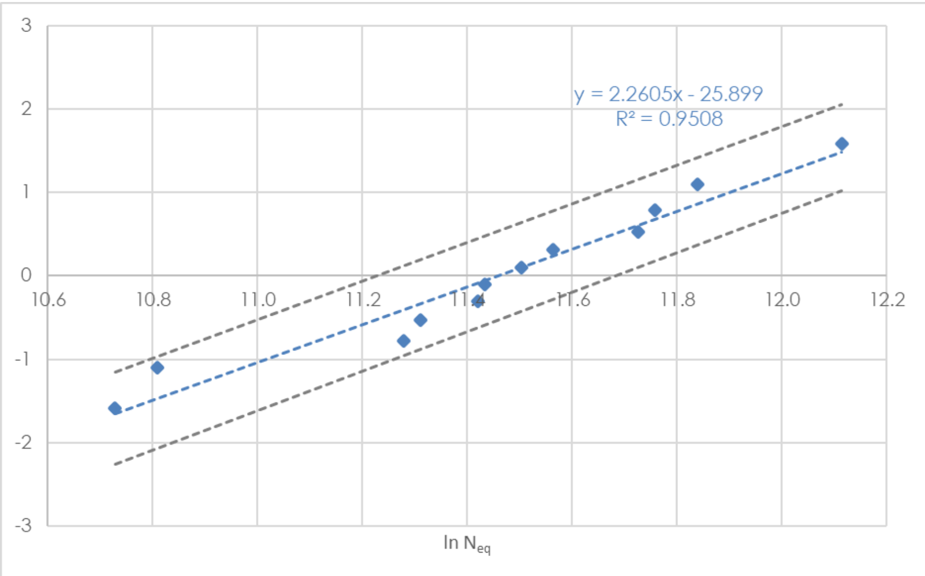


Figure 27: Application of the median ranks method: blue = test results, plain line = trend curve, dashed line = confidence interval [10 % - 90 %].

The Monte Carlo method is applied to estimate the failure probability. The strength values are simulated with the parameters of the 90%-confidence distribution (conservative assumption with respect to the median line). A random sampling of 1 000 000 values is carried out. The failure probability $\widehat{P}_f$, estimated with a statistical tool, is 0.0076. It is below the reliability target (1% over the reference period of 250 000 km). The test therefore demonstrates that the design is reliable with a confidence level of 90%.

Remark 1: When the failure probability is very low and the number of samples is limited, it is recommended to estimate a confidence interval on the Monte Carlo result. The upper boundary of the confidence interval of the failure probability is:

$$L_{sup} = \widehat{P}_f \left(1 + t_{c,N_{MC}-1} \sqrt{\frac{1 - \widehat{P}_f}{N_{MC}\,\widehat{P}_f}} \right)$$

where N_{MC} is the size of the sampling and $t_{c,N_{MC}-1}$ is the quantile of the Student distribution for a degree of freedom $N_{MC} - 1$ and a confidence level c. In this example, N_{MC} = 10^6 and c = 0.9.

Excel ® formula 2013:

0.0076*(1+ T.INV.2T (0.9;10^6-1)*SQRT((1-0.0076)/(10^6*0.0076)))

In the example, L_{sup} is 0.0077. The reliability objective is achieved. Thus, the test shows that the design is reliable at a 90 %-confidence level.

Remark 2: If the field reliability target was not achieved, a new design would have been necessary. If the new design stays close to the old one concerning geometry, material and manufacturing, it is not necessary to perform a failure test to check the reliability, due to the experimental feedback available (σ_{lnR} and b). A time-censored test is sufficient (**Practical sheet 7**).

3.4.5.2 Evaluation of the reliability based on the test results – variable: equivalent speed for a given number of slams (=v_{eq})

For each door, the equivalent speed v_{eq} for $n0$= 80 000 is calculated using the acceleration model. The values v_{eq} are reported in ***Table 17***. The Johnson's rank method (see **Annex 2 - theme 3 - [2], [3]**) is applied to fit a lognormal distribution to the values v_{eq}. The results are given for 3 confidence levels (10, 50 and 90%) in ***Table 18*** and ***Figure 28.***

Table 17: Equivalent speed at* n0 *for the 12 doors tested.

Number of the door	Equivalent speed v_{eq} at n0
#1	1.70
#2	1.65
#3	1.60
#4	1.66
#5	1.73
#6	1.63
#7	1.60
#8	1.49
#9	1.80
#10	1.71
#11	1.63
#12	1.48

Table 18: Strength distribution parameters obtained with the median ranks method.

Confidence level c	10 %	50 %	90 %
μ_R	1.69	1.64	1.59
CV_R	0.058	0.063	0.063

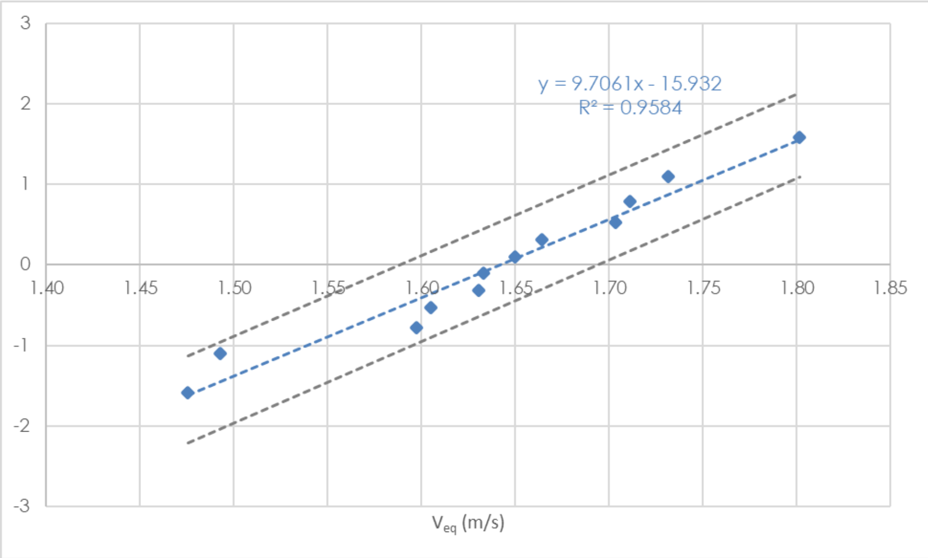


Figure 28: Application of the median ranks method: blue = test results, plain line = trend curve, dashed line = confidence interval [10 % - 90 %].

As in the previous paragraph, the Monte Carlo method is applied to estimate the failure probability. The strength values are simulated with the 90%-confidence distribution. A random sampling with 1 000 000 values is carried out. The estimated failure probability $\widehat{P}_f$ is 0.0078. It is below the reliability target. The test therefore demonstrates that the design is reliable with a confidence level of 90%.

The upper boundary L_{sup} of the confidence interval on the estimated probability is 0.0079. The reliability objective is achieved. The test shows that the design is reliable with a confidence level of 90%.

comment: The estimation of the failure probability obtained with the variable n_{eq} is different than the one obtained with v_{eq}. This is due to the Monte Carlo random sampling and to the distribution hypotheses made for the strength variable (normal distribution for V_{eq} and lognormal distribution for N_{eq}, see footer 9 in **Part B.Ex 2.3.4.4.2**)

PART C. CONCLUSION

The methodology proposed in this handbook allows identification of the phases and input data required to quantify the reliability of components.

The principles and methods proposed in this handbook correspond to the most common and effective practices to enable the stakeholders of the automotive industry to build reliability in a collective and efficient way. Their use must be active rather than passive in order to move towards design for reliability.

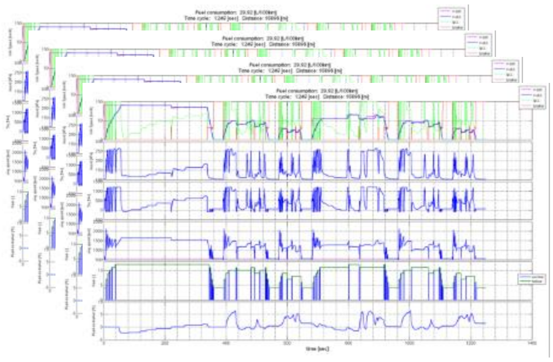
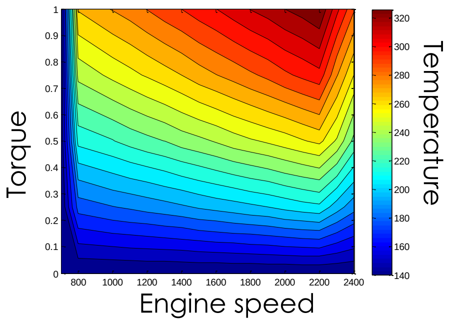
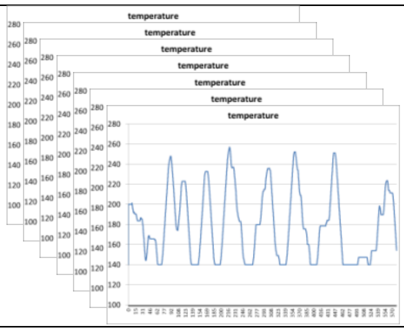
This handbook presents the main proven techniques that can help when building a reliable product. These techniques are detailed through 2 application examples to enable the reader to understand and put into practice the proposed methodology. The practical sheets and the references presented in **Annex 2** give interesting supplements for those wishing to know more about the subject.

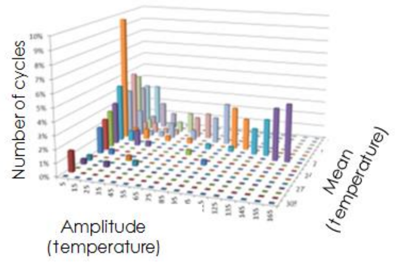
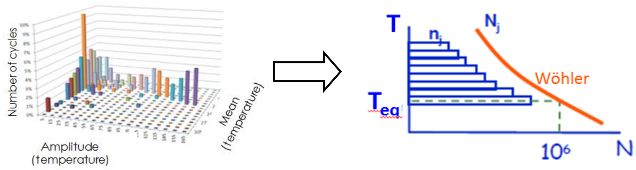
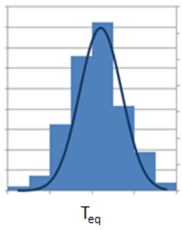
Practical sheet 1 Construction of the load profile of a damaging factor

The load profile of a damaging factor (also called component load profile) is a probabilistic model (distribution, random process) of the loads leading to the component failure. This profile can be built from field data or numerical simulations when the physical quantity representing the load cannot be easily measured among customers.

Example

Let a mechanical component be damaged by thermal fatigue. The damaging factor is the thermal cycling. For obvious cost reasons, carrying out temperature measurements on a large customer panel that is representative of the population is not conceivable. On the other hand, it is possible to build a component load profile from numerical simulation. The steps of the construction of the profile are the following:

1) Measurements and/or simulation of end -users	
	A recording on a few vehicles of some simple physical quantities such as engine speed and torque can be performed. These quantities may also be simulated with an algorithm in order to generate a more significant number of customers.
2) Construction of the transfer function (through Computational Fluids Dynamics (CFD))	
	Temperature values are calculated through CFD for several usage configurations of the engine (engine speed x torque). A transfer function is built based on these calculations. Usage configurations should be wisely selected to guarantee a good precision of the transfer function. Note that the latter is generally built as a polynomial response surface.
3) Computation of the end-user time signals of the damaging factor	
	The transfer function is applied to each end-user recording or simulation of step 1. A time signal in temperature is obtained for each end-user recording or simulation.

<table>
<tr><th colspan="2">4) Rainflow processing of the end-user time signals</th></tr>
<tr><td>
</td><td>For each end-user time signal, the temperature cycles are extracted using a Rainflow procedure [1]. Each cycle is characterized with a mean value and an amplitude. The Rainflow result is generally depicted as a 3D histogram called Rainflow matrix (see opposite). It provides the number of cycles in each {mean x amplitude} bin. The calculation of the matrix is realized using a data analysis tool [1].</td></tr>
<tr><th colspan="2">5) Summarising of the customer Rainflow matrices to synthetic values</th></tr>
<tr><td>
</td><td>A synthetic value is calculated for each end-user Rainflow matrix. This value obtained by applying the Miner's rule [2] can be: an equivalent temperature T_{eq} at a given number of cycles, an equivalent number of cycles N_{eq} at a given temperature or a damage. On the left, the value T_{eq} at 10^6 cycles is obtained.</td></tr>
<tr><th colspan="2">6) Assessment of the component load profile</th></tr>
<tr><td>
</td><td>A statistical distribution is fitted to the data set of synthetic values (see fitting examples in Practical sheet 3). It defines the component load profile.</td></tr>
</table>

Remark: The different steps to implement depend on the type of measured loads:

1) If it is possible to measure the damaging factor and to have a single value per end-user, the profile can be assumed directly. The approach then consists in steps 1 and 6 (example: number of openings / closings of a door).
2) If time measurements of the damaging factor can be performed on a customer panel representative of the general population, it is not necessary to realize the first 3 steps.
3) If the measured quantity is not the damaging factor, all the steps must be realized.

Practical sheet references

[1] *Standard practices for cycle counting in fatigue analysis* - ASTM E 1049-85. (Reapproved 2005) - ASTM International.

[2] *La fatigue des matériaux et des structures* - C. Bathias, J.P. Baïlon - Hermès-Lavoisier, 2nde édition - 1997.

Normal (or Gaussian) Distribution

Main phenomena modeled with a normal distribution:
Variability of the limit strength, variability of the fatigue strength at a given number of cycles, variability of a geometrical dimension or of biometric characteristics such as height and weight.

Parameters:
Mean: $\mu = E[X]$ and standard deviation: $\sigma = \sqrt{Var(X)}$

Expressions:

- Probability density function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$
- Cumulative distribution function: $F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right) dt = \Phi\left(\frac{x-\mu}{\sigma}\right)$ where Φ is the standard normal cumulative distribution function ($\mu = 0$ and $\sigma = 1$).

Example:
A normal distribution is fitted to 100 values of the fatigue strength of a drive shaft at a given number of cycles (***Figure 29***). The fatigue strength is expressed as a torsion torque whose mean value is $\mu = 173.99$ daN.m and standard deviation $\sigma = 14.99$ daN.m. The probability that the drive shaft fails when a torsion torque lower or equal to 150 daN.m is applied is:

$$F(150) \approx 0.055$$

Excel ® Formula 2013: NORM.DIST(150;173.99;14.99;TRUE)

The normal probability plot is used to visually assess the goodness of fit of the normal distribution to the data.

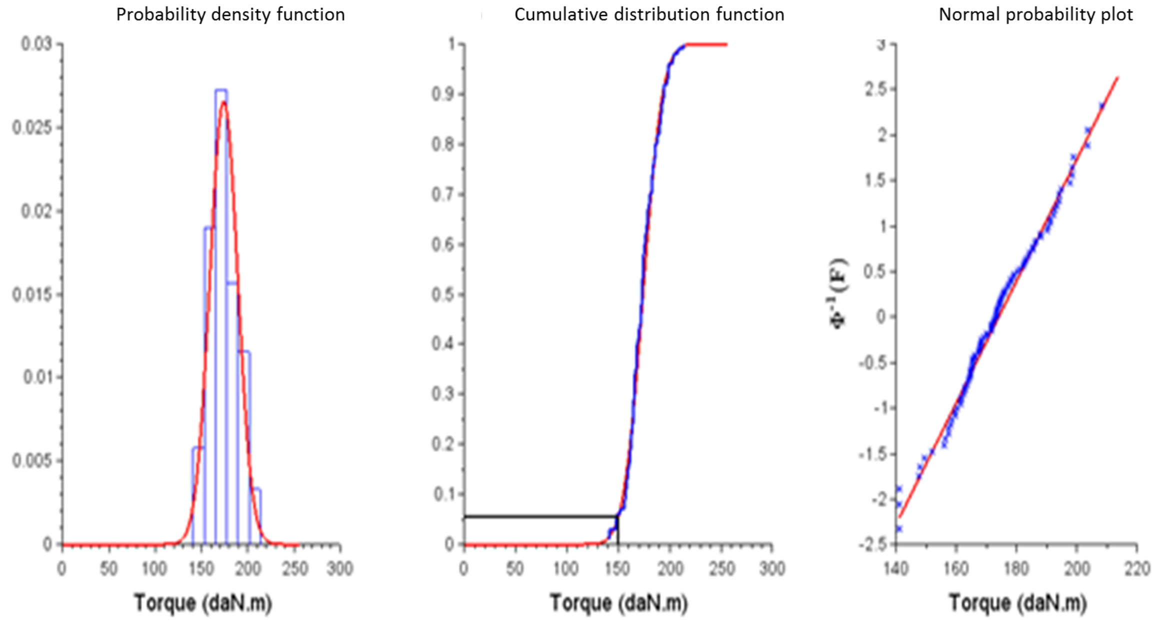


Figure 29: Normal distribution fitted to 100 values of the fatigue strength of a drive shaft.

Lognormal distribution

Main phenomena modeled with a lognormal distribution:

Mileage, stress variability, variability of the number of cycles to failure in the high cycle fatigue domain with finite life, variability of any positive degradation such as wear [1].

Parameters: If X follows a lognormal distribution then $\ln X$ is a normal distribution which parameters are:

$$\mu_{\ln X} = \ln(E[X]) - 0.5 \ln\left(1 + \frac{Var(X)}{E[X]^2}\right) \text{ and } \sigma_{\ln X} = \sqrt{\ln\left(1 + \frac{Var(X)}{E[X]^2}\right)}$$

Expressions for $E[X]$ and $Var(X)$ using these parameters:

$$E[X] = \exp\left(\mu_{\ln X} + \frac{\sigma_{\ln X}^2}{2}\right) \text{ and } Var(X) = (\exp(\sigma_{\ln X}^2) - 1)\exp(2\mu_{\ln X} + \sigma_{\ln X}^2)$$

Expressions:

- Probability density function: $f(x) = \frac{1}{x\,\sigma_{\ln X}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu_{\ln X}}{\sigma_{\ln X}}\right)^2\right)$
- Cumulative distribution function: $F(x) = \Phi\left(\frac{\ln x - \mu_{\ln X}}{\sigma_{\ln X}}\right)$

Example:

A lognormal distribution is fitted to 100 values of customer monthly mileage (***Figure 30***). The mean of the logarithm of the monthly mileage is $\mu_{\ln X} = 7.01$ and its standard deviation $\sigma_{\ln X}$ is 0.564. The probability that a end-user covers more than 3 000 km in a month is:

$$1 - F(3\ 000) \approx 0.032$$

Excel ® Formula 2013: 1-LOGNORM.DIST(3000;7.01;0.564;TRUE)

The probability plot shows the probability model fits the data well.

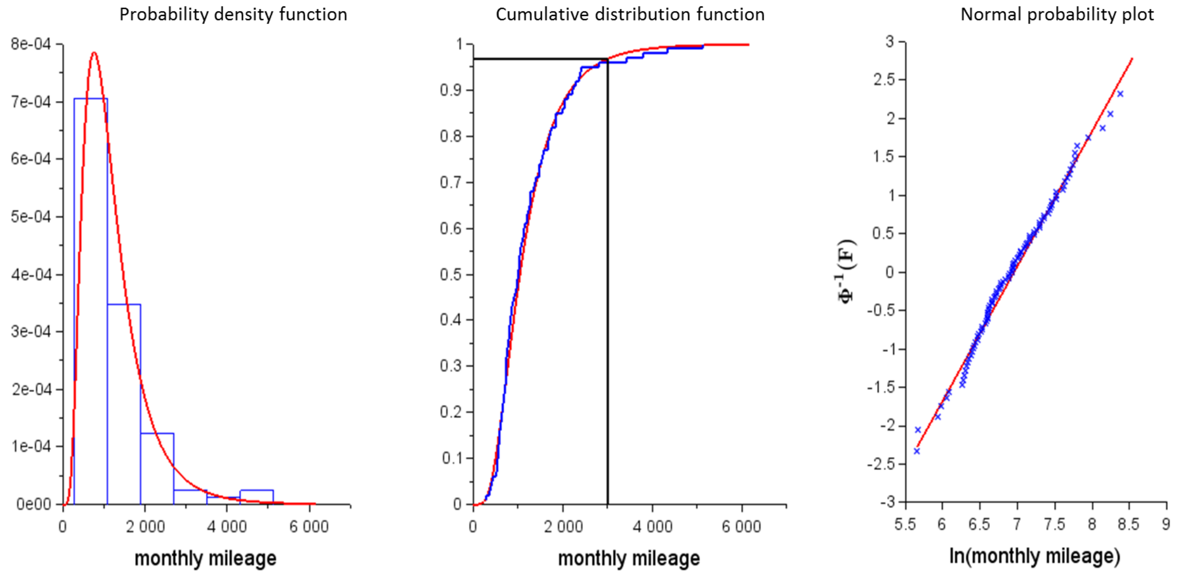


Figure 30: Lognormal distribution fitted to 100 monthly mileage values.

Weibull distribution

Main phenomena modeled with a Weibull distribution:

Variability of time to failure for the different life periods of a component (see ***Figure 2***): early life: $\beta < 1$ / useful life: $\beta = 1$ / wear-out: $\beta > 1$, variability of the degradation (wear). The Weibull distribution is particularly recommended for modeling the wear phenomenon.

Parameters: Shape parameter β, scale parameter η, location parameter γ such that:

$$\begin{cases} E[X] = \eta\,\Gamma\left(1 + \frac{1}{\beta}\right) + \gamma \\ Var\ (X) = \eta^2\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right] \end{cases}$$

Figure 31 gives an extract of the Gamma function tables (Γ).

Expressions:

- Probability density function (***Figure 32***): $f(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right]$
- Cumulative distribution function: $F(x) = 1 - \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right]$ and
- Reliability function: $R(x) = 1 - F(x) = \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right]$

x	Γ (x)	x	Γ (x)
1	1	2	1
1.05	0.9735	2.05	1.0222
1.1	0.9514	2.1	1.0465
1.15	0.9330	2.15	1.0730
1.2	0.9182	2.2	1.1018
1.25	0.9064	2.25	1.1330
1.3	0.8975	2.3	1.1667
1.35	0.8912	2.35	1.2031
1.4	0.8873	2.4	1.2422
1.45	0.8857	2.45	1.2842
1.5	0.8862	2.5	1.3293
1.55	0.8889	2.55	1.3777
1.6	0.8935	2.6	1.4296
1.65	0.9001	2.65	1.4852
1.7	0.9086	2.7	1.5447
1.75	0.9191	2.75	1.6084
1.8	0.9314	2.8	1.6765
1.85	0.9456	2.85	1.7494
1.9	0.9618	2.9	1.8274
1.95	0.9799	2.95	1.9108
		3	2

Figure 31: Part of the reference table of the Gamma function.

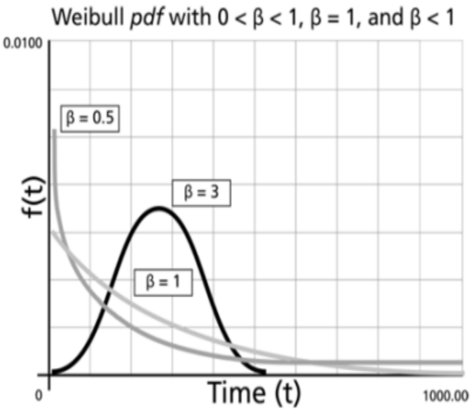


Figure 32: Weibull probability distribution function (γ = 0) for different values of β.

Example:

A Weibull distribution is fitted to 100 lifetime values of a starter (***Figure 33***). The lifetime is expressed as the number of activations to failure. The parameters of the Weibull distribution are β = 4.89, η = 356,882 and γ = 0. The probability that a starter fails before 210 000 activations is:

$$F(210\,000) \approx 0.087$$

Excel ® Formula 2013: WEIBULL.DIST(210000;4,89;356882;TRUE)

The Weibull distribution is often represented on an Allan-Plait paper which allows to visually check if the distribution fits the data well (see the Weibull plot on ***Figure 33***).

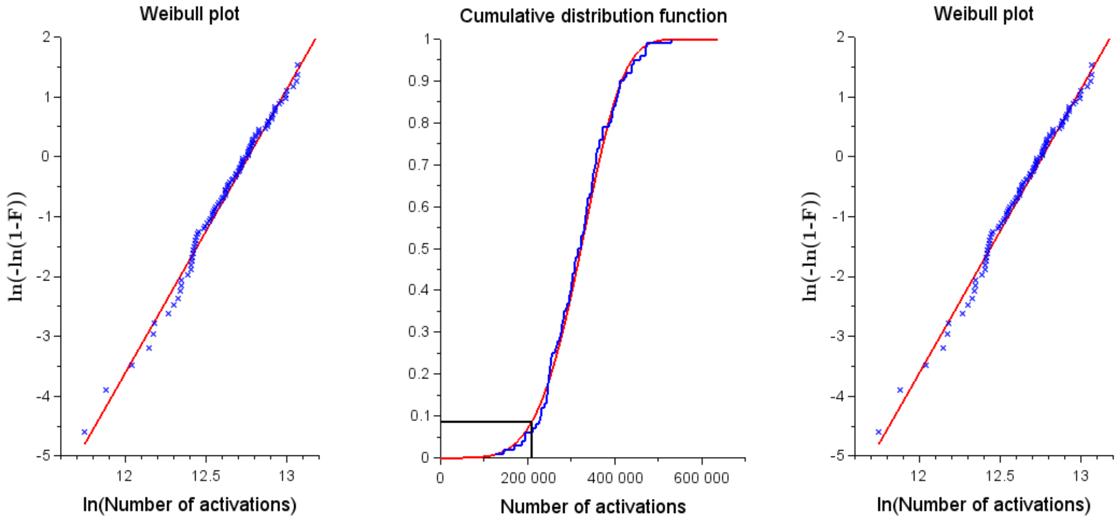


Figure 33: Weibull distribution fitted to 100 lifetime values of a starter.

Exponential Distribution

Main phenomena modeled with an exponential distribution:

The exponential distribution is used to model the random failures during the service life of electronic components. It is the particular case $\beta = 1$of the Weibull distribution (***Figure 32***).

Parameters: Failure rate λ, location parameter γ so that:

$$E[X] = \gamma + \frac{1}{\lambda} \text{ et } Var(X) = \frac{1}{\lambda^2}$$

Expressions:

- Probability density function: $f(x) = \lambda \exp[-\lambda(x-\gamma)]$
- Cumulative distribution function: $F(x) = 1 - \exp[-\lambda(x-\gamma)]$
- Reliability function: $R(x) = 1 - F(x) = \exp[-\lambda(x-\gamma)]$

The exponential distribution is often depicted by its reliability function. This distribution is mainly used for electronic components.

Example:

An exponential distribution is fitted to 100 lifetime values (in hours) of an electronic component (***Figure 34***). Its parameters are $\lambda = 1.01 \times 10^{-5}$ and $\gamma = 0$. The probability that a component fails after 350 000 hours is:

$$1 - F(350\ 000) \approx 0.029$$

Excel Formula ® 2013: 1-EXPON.DIST(350000;0,0000101;TRUE)

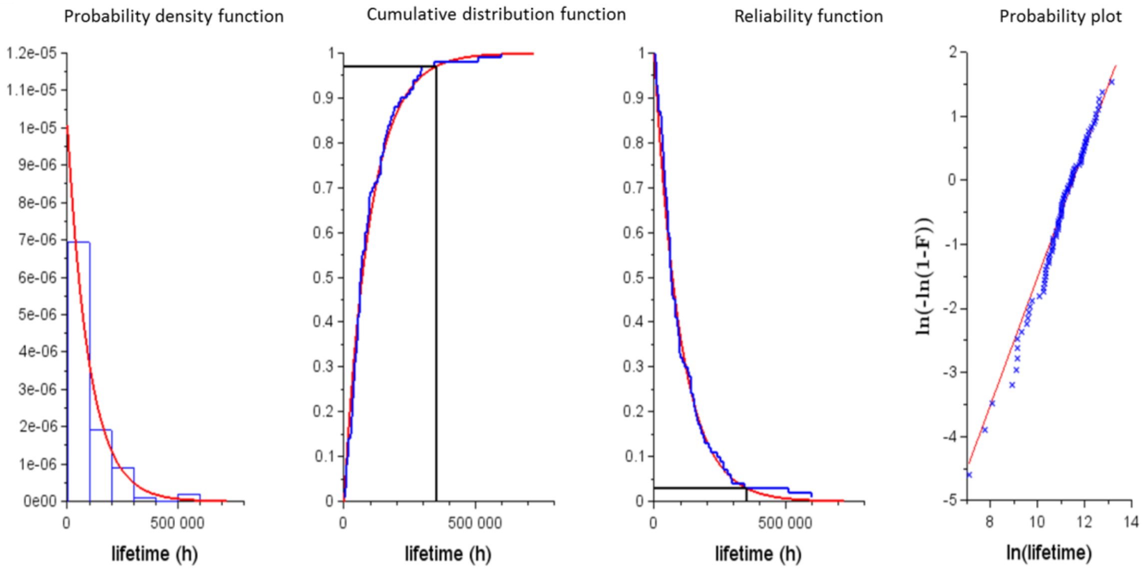


Figure 34: Exponential distribution fitted to 100 lifetime values of an electronic component.

Practical sheet reference

[1] *Produits métalliques - Essais de fatigue - Traitement statistique des données* - AFNOR A 03-405 - 1991.

Failure data may be obtained from testing or end-user feedback. The statistical analysis of these data aims at defining the distribution of the quantity measuring the reliability (mileage, number of cycles, number of activations). Different methods may be applied for that (references [1,2]): Johnson's rank method, median ranks, maximum likelihood estimation and hazard-plotting. The Practical sheet 3 illustrates some of these methods on 2 examples: one with experimental data and one with field data.

Test results analysis

As illustrated in ***Figure 35***, a test result can be complete or non-complete:

- Complete data: the time to failure is known at the end of the test.
- Non-complete data: the time to failure is not known at the end of the test.

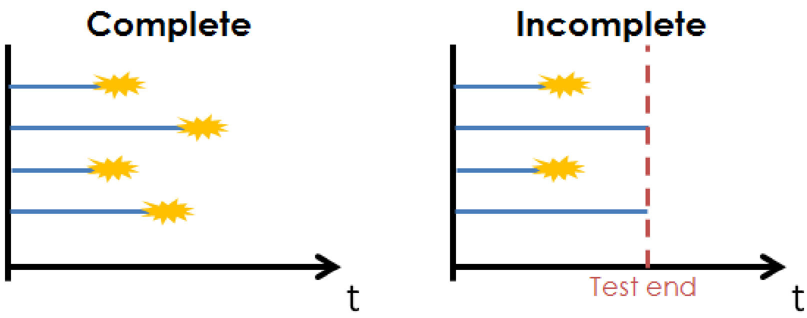


Figure 35: Examples of complete and incomplete test data. The blue segment represents test duration; the failure is depicted in yellow. For the example with incomplete data, 2 components have not failed by the end of the test. These results are said to be right censored.

When all Test data is complete, the median rank method is generally used.

When some data is incomplete, the Johnson rank method can be used in the case where the number of incomplete data is small. In cases where the number of incomplete data is high, the Maximum Likelihood method or the hazard plotting method are generally used.

Example:

10 components are tested to failure. The results are reported in ***Table 19***. The median ranks method of is applied to determine the cumulative distribution function $F_C(n)$ of the number of cycles to failure and the confidence interval.

Table 19: Application of the median ranks method on complete data.

tests in ascending order	Rang r_i	$x_i = \ln n_i$	Log-normal distribution $Y[F_{50\%}(n_i)]$	Log-normal distribution $Y[F_{10\%}(n_i)]$	Log-normal distribution $Y[F_{90\%}(n_i)]$	Weibull distribution $Y[F_{50\%}(n_i)]$	Weibull distribution $Y[F_{10\%}(n_i)]$	Weibull distribution $Y[F_{90\%}(n_i)]$
62200	1	11,04	-1,50	-2,31	-0,82	-2,67	-4,55	-1,47
79600	2	11,28	-0,99	-1,60	-0,42	-1,73	-2,88	-0,89
99000	3	11,50	-0,65	-1,20	-0,13	-1,21	-2,09	-0,52
107680	4	11,59	-0,37	-0,89	0,13	-0,82	-1,57	-0,22
108210	5	11,59	-0,12	-0,62	0,37	-0,51	-1,17	0,04
191270	6	12,16	0,12	-0,37	0,62	-0,23	-0,83	0,28
193240	7	12,17	0,37	-0,13	0,89	0,03	-0,52	0,52
200815	8	12,21	0,65	0,13	1,20	0,30	-0,22	0,77
273320	9	12,52	0,99	0,42	1,60	0,60	0,08	1,07
303400	10	12,62	1,50	0,82	2,31	0,99	0,46	1,52
		Least Squares results	**$\mu_{\ln N}$=11,87**	**$\mu_{\ln N}$=12,20**	**$\mu_{\ln N}$=11,53**	**β=2,01**	**β=2,66**	**β=1,66**
			$\sigma_{\ln N}$=0,60	**$\sigma_{\ln N}$=0,58**	**$\sigma_{\ln N}$=0,58**	**η=185363**	**η=235457**	**η=133683**
			r^2 (correlation)=0,95			**r^2 (correlation)=0,93**		

Excel ® Formula 2013:

Log-normal distribution $Y[F_{50\%}(ni)]$: NORM.S.INV(BETA.INV(c;ri;N-ri+1))

Weibull distribution $Y[F_{50\%}(ni)]$: LN(LN(1/(1-BETA.INV(c;ri;N-ri+1))))

with c = 50 % and N = 10.

A log-normal distribution and a Weibull distribution are tested (***Figure 36***). The correlation coefficient r^2 is used to determine the distribution that best fits the data. The log-normal distribution is the most appropriate (0.95>0.93).

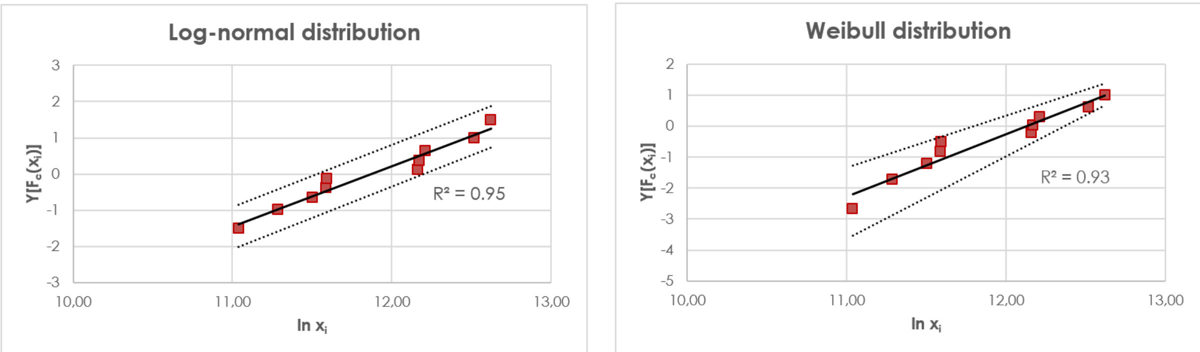


Figure 36: Adjustment of a linear relation on the data after the change of variables.

By experience, It is recommended to have at least 7 failures to fit a Weibull model; this gives usable confidence intervals. Whatever the number of failures, the confidence interval of the cumulative distribution function must be calculated.

Input data:

- Test results (failures observed or degradations measured on track or during testing)

Output data:

- Statistical modeling of the physical damaging phenomenon statistical model and estimation of its parameters
- Statistical distribution (Weibull) modeling failures observed during test
- Mean level of degradation and test scatter

Analysis of field failure data

To estimate the field failure probability, it is necessary to know the age of the vehicles at failure but also the number of vehicles which have not yet failed, as well as their ages at the date of observation. The field failure data should thus be completed with monthly delivery figures (sales, production…).

Example:

97 failures of a component have been noted among the customers since the introduction of a new manufacturing process 23 months ago. For each failure, the age of the component (in months) is known. Date of comissionning of the last 23 months are also known (***Figure 37***). Maximum likelihood estimation [1], [2] is applied to fit a Weibull distribution to the data. The parameters β and η and the cumulative distribution function are obtained for a given confidence level.

Entry into service data		
Production month	Quantity	Age (months)
janv.-14	2133	23
févr.-14	2934	22
mars-14	2663	21
avr.-14	2519	20
mai-14	2257	19
juin-14	2756	18
juil.-14	2482	17
août-14	2848	16
sept.-14	3101	15
oct.-14	3357	14
nov.-14	3071	13
déc.-14	2897	12
janv.-15	2591	11
févr.-15	2835	10
mars-15	3038	9
avr.-15	3041	8
mai-15	2739	7
juin-15	2969	6
juil.-15	2983	5
août-15	2981	4
sept.-15	2793	3
oct.-15	2735	2
nov.-15	2756	1
Total	64480	

Customer failures	
Number of failures	Age (months)
8	22
12	21
14	19
13	13
17	16
12	10
13	7
7	5

Maximum likelihood estimation

	10%	50%	90%
β	2.82	3.12	3.46
η	97	118	144

Cumulative distribution function

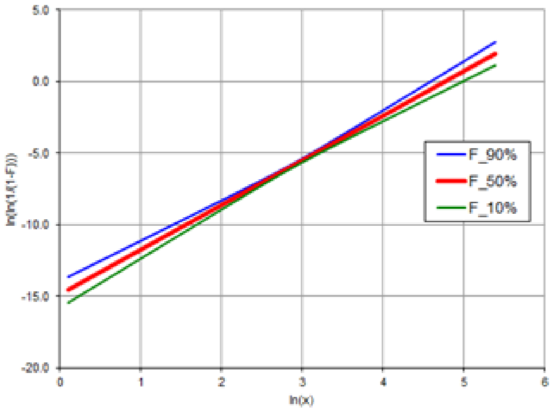


Figure 37: Application of the maximum likelihood estimation to failure and entry into service data.

Practical sheet references

[1] *Weibull Analysis Handbook* - R. B. Abernethy, J. E. Breneman, C. H. Medlin, G. L. Reinman - 1983. Available at: http://www.dtic.mil/dtic/tr/fulltext/u2/a143100.pdf

[2] *Warranty Data Collection and Analysis, Springer Series in Reliability Engineering* - W. R. Blischke, M. Rezaul Karim, D. N. Prabhakar Murthy - 2011.

[3] *Modeling Market Incident Rate Using Weibull Distribution* – L. Bonvin, M NDiaye, C. Ramus-Serment, N. Forissier, B. Regis, R. Laronde, C, Niggel – SIA – 2020

[4] *Modeling Market Incident Levels (Warranty & Over) using Weibull Distribution (Part 2)* – L. Bonvin, C. Ramus-Serment, N. Forissier, B. Regis, R. Laronde, C, Niggel – SIA – 2022

The principle of an accelerated test is to subject a component to conditions that are more severe than those occurring during normal customer usage in order to reproduce the failure in a shorter amount of time. Normal customer usage is linked to the test severity through the acceleration model. A few common laws are introduced in this practical sheet.

An accelerated test should neither create new failure modes nor modify the basic mechanisms leading to failure.

Acceleration model for mechanical fatigue: Basquin's relation

The most common acceleration model for mechanical fatigue is the Basquin's relation which is illustrated in ***Figure 38*** and whose expression is:

$$N\,S^b = B$$

where N is the number of cycles to failure, S is the stress level, B and b are some constants that are characteristics of the material. Their values are determined through testing or reported in data bases [1], [2], [3] (a value of 8 is often used for the parameter b of aluminum alloys).

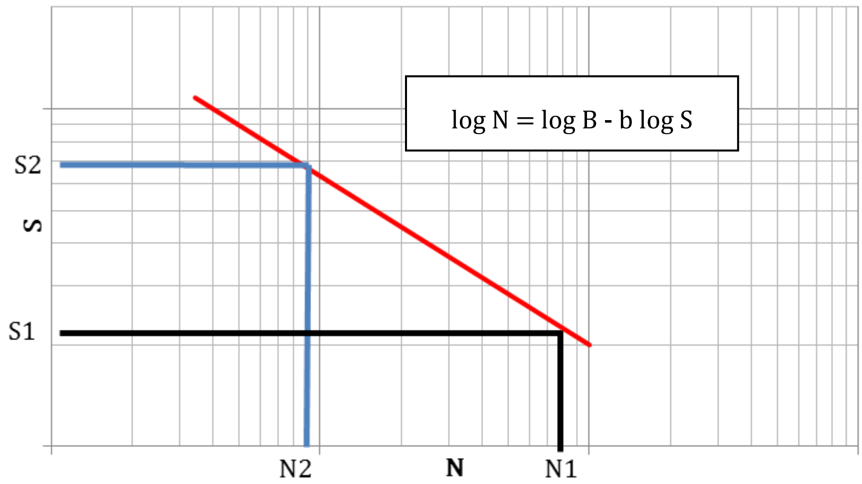


Figure 38: Application of the Basquin's relation. ***(S1, N1) corresponds to the normal customer usage, (S2, N2) to the accelerated test.***

The normal customer usage is characterized by a stress level $S1$ repeated $N1$ cycles. The accelerated test stress $S2$ is chosen in order to reduce the number of cycles ($N2<N1$) without modifying the physical failure mechanism. The Basquin's relation (in red) enables to determine the number of cycles $N2$:

$$\begin{matrix} B = N1\,S1^b \\ N2 = B\,S2^{-b} \end{matrix} \Leftrightarrow N2 = N1\left(\frac{S1}{S2}\right)^b$$

The damage induced by $N2$ cycles at $S2$ is equal to the damage produced by $N1$ cycles at $S1$.

Acceleration model for thermal cycling: Coffin-Manson's relation

A possible acceleration model for thermal cycling is the Coffin-Manson's relation which is illustrated in ***Figure 39*** and whose expression is:

$$N\,\Delta T^b = B$$

where N is the number of cycles to failure, ΔT the thermal cycle range and B and b some constants that are characteristics of the material and test ($b \approx 2$ for a braze-welded joints [4]).

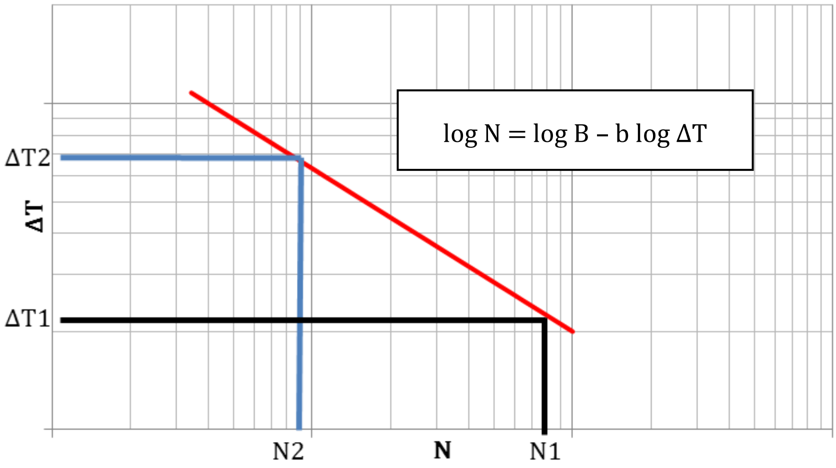


Figure 39: Application of the Coffin-Manson's relation. $(\Delta T1, N1)$ ***corresponds to the normal customer usage,*** $(\Delta T2, N2)$ ***to the accelerated test.***

The normal customer usage is characterized by a thermal cycle $\Delta T1$ and a number of cycles to failure $N1$. The accelerated test thermal cycle $\Delta T2$ is chosen in order to reduce the number of cycles ($N2<N1$) without modifying the physical failure mechanism. The Coffin-Manson's relation (in red) enables to determine the number of cycles $N2$:

$$\begin{aligned} B &= \frac{N_1}{(\Delta T1)^{\gamma}} \\ N2 &= \frac{B}{(\Delta T2)^{\gamma}} \end{aligned} \Leftrightarrow N2 = N1 \left(\frac{\Delta T2}{\Delta T1}\right)^{\gamma}$$

Acceleration model for thermochemical degradation: Arrhenius' relation

The most common acceleration law for thermochemical degradation (*e.g.* corrosion, creep…) is the Arrhenius' relation which is illustrated in ***Figure 40*** and whose expression is:

$$t = B \exp\left(\frac{E_a}{k_B\, T}\right)$$

where t is the exposure time of the component to the temperature T (in K), B a constant, k_B the Boltzmann constant (8.62×10^{-5} eV.K^{-1}) and E_a the activation energy (≈1 eV) which depends on the material. This latter depends also on the operating temperature range. It is generally assumed that the activation energy is constant when the temperature range is not too large.

The normal customer usage is characterized by a temperature $T1$ applied during a time $t1$. The accelerated test temperature $T2$ is chosen in order to reduce the exposure time ($t2<t1$) without modifying the physical failure mechanism. The Arrhenius' relation (in red) enables to determine the time $t2$:

$$\begin{aligned} B &= t1 \exp\left(-\frac{E_a}{k_B\, T1}\right) \\ t2 &= B \exp\left(\frac{E_a}{k_B\, T2}\right) \end{aligned} \Leftrightarrow t2 = t1 \exp\left(\frac{E_a}{k_B}\left(\frac{1}{T2} - \frac{1}{T1}\right)\right)$$

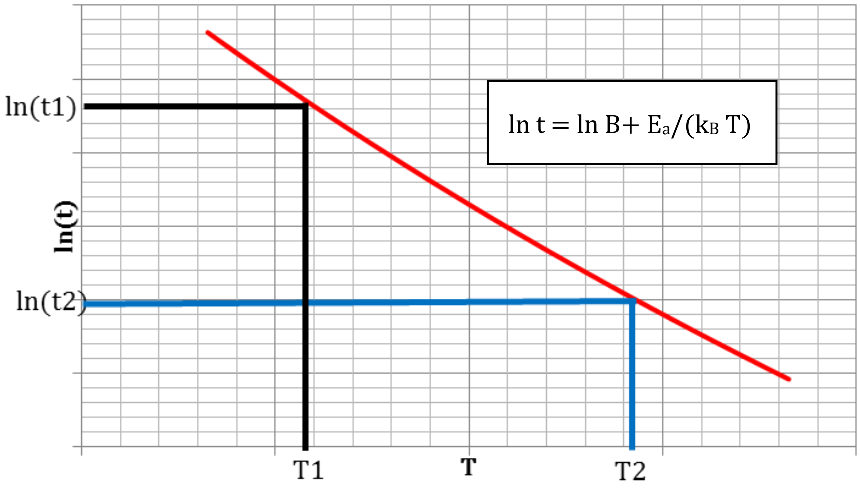


Figure 40: Application of the Arrhenius' relation. (T1, t1) = ***normal customer usage,*** (T2, t2) = ***accelerated test.***

Acceleration models for other phenomena

Acceleration models for other phenomena may be found in literature [5]. For example: Peck's relation for humidity and the Norris-Landzberg's relation which is often used to model cracks in welds for electronic. The principle of these models is similar to the examples above.

Practical sheet references

[1] *Guide d'application de la démarche de personnalisation en environnement mécanique* - PR NORMDEF 0101 - Edition 01 - 2009.

[2] *Préconisations pour les caractéristiques statistiques de résistance en fatigue des tôles en acier* – SIA Handbook – DC-05-01 - 2017.

[3] *Produits métalliques - Essais de fatigue - Traitement statistique des données* - AFNOR A 03-405 - 1991.

[4] JEDEC Publication n°112E p.48.

[5] *Accelerated testing: Statistical models, test plans and data analysis* - W. B. Nelson - 2004.

Practical sheet 5: Test adjustment with Test/Field comparison

The Test/Field comparison methods (Weibull curves or analysis of the degradation) allow to readjust the existing acceptance criteria of tests (physical and virtual) using the field failure data. This practical sheet presents the 2 comparison methods mentioned above and give some examples.

Comparison of Test/Field Weibull curves

The adjustment of the tests by comparing the Test/FieldWeibull curves consists in 3 steps:

1. Modeling of the field data failure by a Weibull distribution and evaluation of the field reliability.
2. Modeling of the test results with a Weibull distribution.
3. Evaluation of the ratio of the field reliability target to the reliability assessed with field data. Adjustment of the test acceptance criterion by multiplying it by the ratio found previously.

Application to a turbocharger:

Input data:

- The field failure data of a turbocharger are presented in ***Figure 41***. 2 failure modes are observed: early failure (problem of process/manufacturing creating a balance defect) at small mileage and wear-out failures (here, a hot creep of the turbo blades). For the wear-out failures, only 0.5 % of the vehicles fails at 50 000 km.
- Test results of turbochargers rig are given in ***Figure 42***. The number of hours after which 10% of failure can be observed is: B10 = 200h (see **Part A.5.4.1**). It has been checked that the test reproduces the same wear-out mode.
- Reliability objective: 0.5 % of failures at 100 000 km (in orange on ***Figure 41***).

Remark: numerical data of this application comes from a study and cannot be recalculated.

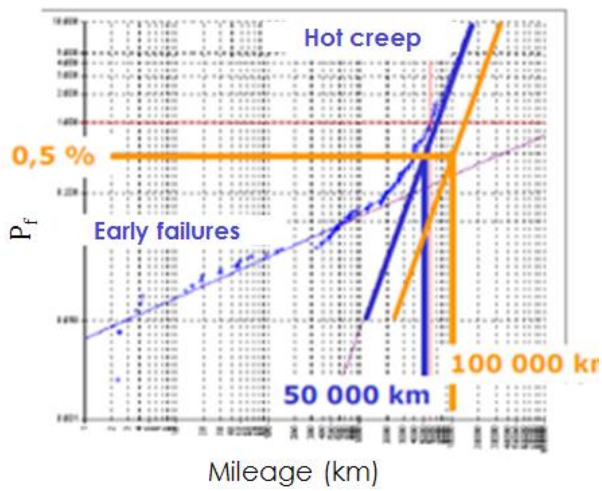


Figure 41: Field failures (blue points) drawn in Allan-Plait paper (failure probability in terms of mileage).

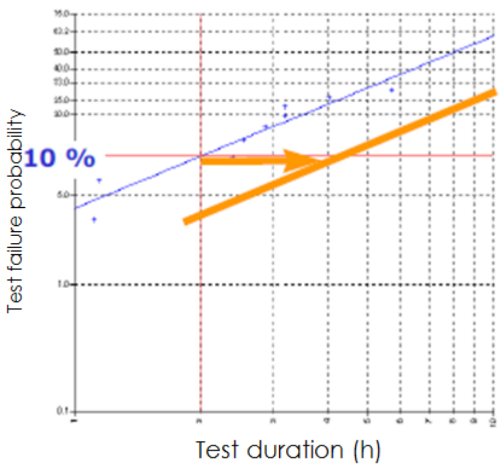


Figure 42: Test failures (blue points) drawn in Allan-Plait paper (failure proportion in terms of the test duration).

Step 1:

A Weibull distribution is fitted for each type of failure (thin blue line = early failure, thick blue line = hot creep in ***Figure 41***). In this example, 0.5 % of the vehicles have failed by 50 000 km for the wear-out. The reliability target is not reached: 50 000 km <100 000 km.

Step 2:

A Weibull distribution is fitted to the test results lifetime in order to estimate the quantile at 10 % called $\mathrm{B10}$. In this example, the $\mathrm{B10}$ is 200 h. Before getting the end-user feedback, it was supposed that a 200h test could allow to reach 0.5 % of failure at 100 000 km.

Step 3:

The ratio between the reliability objective (0.5 % at 100 000 km) and the customer reliability observed (0.5 % at 50 000 km) is 100 000 / 50 000 = 2. Assuming that test and field lifetimes are proportional, the feedback allows to define the new test acceptance criterion (in orange on ***Figure 42***):

$$\mathrm{B10} = 200\ \mathrm{h} \times \frac{100\,000}{50\,000} = 400\ \mathrm{h}$$

The test must verify that there is no more than 10 % of failures after 400 hours.

Remark: It is not mandatory to use a Weibull distribution to model the data. A lognormal distribution can be used as well.

Comparison of Test/Field degradations

The adjustment of the test, by comparing test and field degradations, is feasible only when the degradation phenomenon is measurable (example: wear-out). There are 3 steps:

1. Modeling of field degradation measurements and evaluation of the field reliability.
2. Modeling of accelerated test degradation measurements.
3. Evaluation of the acceleration factor of the test. Calibration of the field degradation distribution for meeting the reliability objective and adjustment of the test using the acceleration factor.

Application to a component subject to wear:

Input data:

- Wear measurements performed on a sample of components collected among end-users. The sample is assumed to be representative of the whole population.
- Wear measurements performed after a test on a circuit assumed to be representative of the customer wear phenomenon. The test is accelerated in comparison with customer usage. The acceleration factor of the test will be estimated in step 3.
- A failure is defined as a wear greater than 3 mm.
- Field reliability target: 10 % of failures at 200 000 km.

Remark: The numerical data of this application come from a study and cannot be recalculated.

Step 1:

Firstly, field wear measurements are extrapolated linearly to the reference period 200 000 km using a relation between the degradation level and mileage (see **Practical sheet 8**). Secondly, a statistical distribution is fitted to the wear values at 200 000 km (***Figure 43***) in order to assess the field failure probability (failure = wear > 3 mm). Its value is 15 %. Thus the objective of 10 % failure at 200 000 km is not achieved. In this example, the mean field wear value at 200 000 km observed is 1.25 mm.

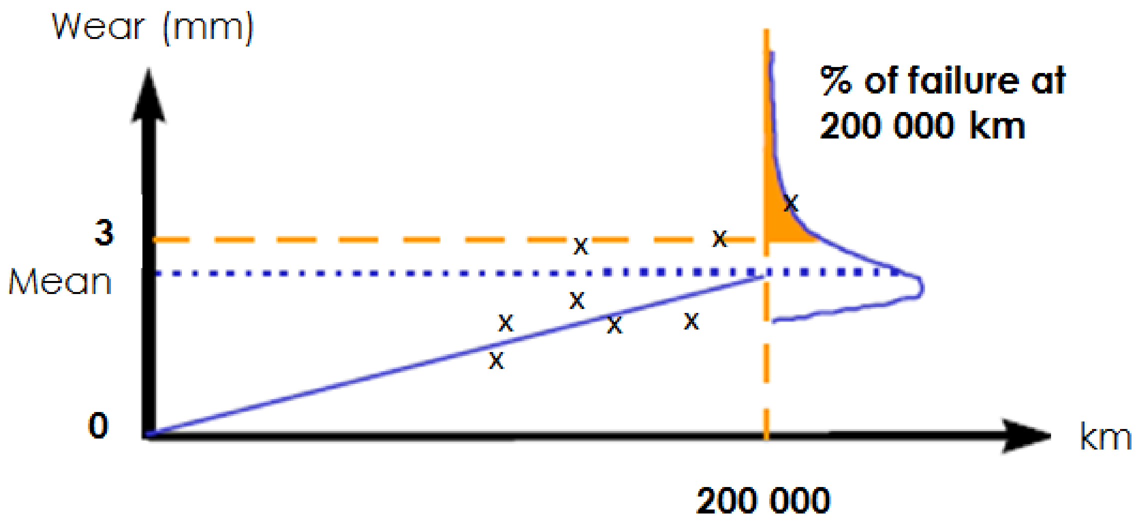


Figure 43: Estimation of the field failure probability at 200 000 km. The crosses correspond to wear measurements of customer components.

Step 2:

Wear measurements of the accelerated test are extrapolated at 200 000 km. The study shows that the mean test wear value at 200 000 km is 2.5 mm.

Step 3:

Firstly, the acceleration factor is estimated as the ratio of the mean test wear at 200 000 km to the mean field wear at 200 000 km. It is 2.5 / 1.25 = 2. Therefore, 100 000 km of test reproduce the same wear as 200 000 km of customer usage on average.

Secondly, the position of the field wear distribution at 200 000 km is calibrated in order to meet the reliability target (10 % of failures at 200 000 km). For that purpose, the mean value is recalculated assuming the standard deviation of this distribution is the same as the one found at step 1 (an hypothesis on the coefficient of variation can also be made). The study shows that the mean customer wear at 200 000 km, allowing to meet the reliability objective, is 0.75 mm (see ***Figure 44***):

$$F(3; \mu; \sigma_{\text{step 1}}) = P_f = 10\% \ \rightarrow \mu \approx 0.75 \text{ mm}$$

where F is the cumulative distribution function (same type than the one determined in step 1) and 3 is the wear failure threshold in mm.

Thirdly, the test is readjusted using the acceleration factor of the test. Knowing that the mean field wear at 200 000 km must be 0.75 mm, the acceptable mean test wear at 200 000 km is 0.75 x 2 = 1.5 mm (or 0.75 mm at 100 000 km).

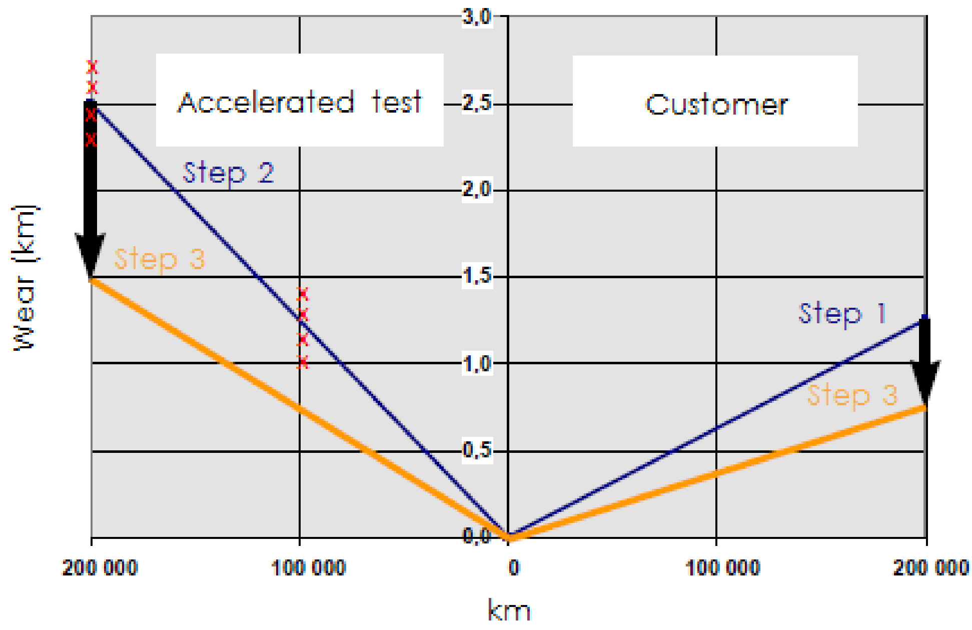


Figure 44: Adjustment of the accelerated test after calibration of the mean wear at 200 000 km (1.25 mm ⇨ 0.75 mm). Red crosses correspond to preliminary test measurements. Blue segments correspond to the input data. Orange segments indicate the adjustment made to meet the reliability target.

Practical sheet 6: Reliability assessment using the Stress-Strength method

The Stress-Strength method [1-3], depicted in ***Figure 45***, is used to quantify the reliability of a component. It is based on the comparison of 2 distributions:

- The stress C which represents the scatter of the load that is applied to the component (variabilities of end-user severity, environmental conditions…).
- The strength R which characterizes the scatter of the mechanical behavior of the component (variabilities of the geometrical dimensions, material properties, manufacturing process…).

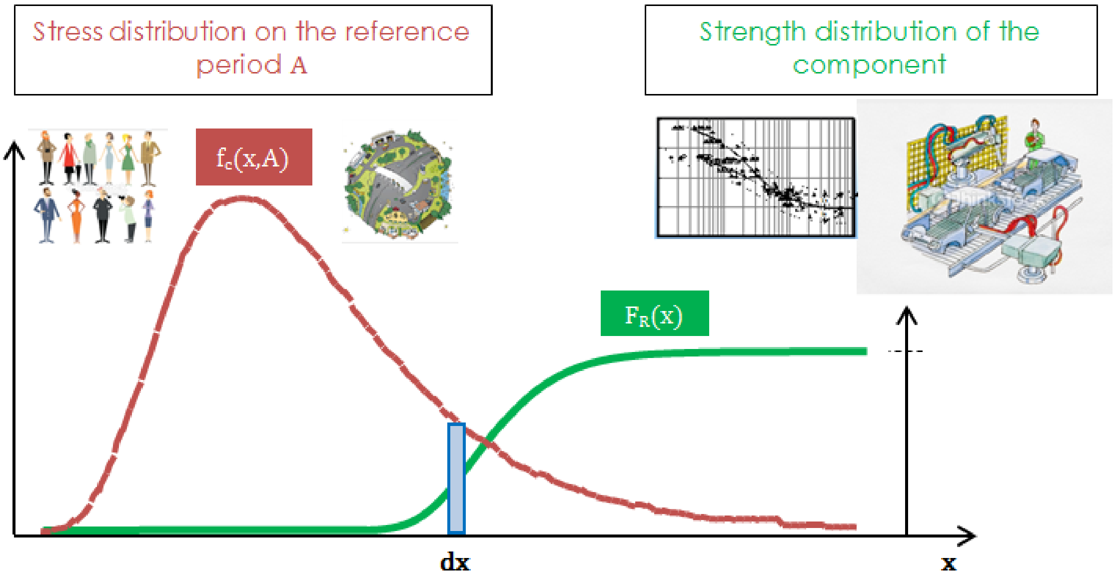


Figure 45: Stress-Strength method.

The component fails if the strength is lower than the stress. The failure probability P_f on the reference period A is then:

$$P_f = \text{Prob}(R \leq C) = \int_{-\infty}^{+\infty} F_R(x).f_C(x,A)dx$$

where $f_C(x,A)$ is the probability distribution function of the stress variable and $F_R(x)$ the cumulative distribution function of the strength variable.

The failure probability is assessed through numerical integration. In some cases, the failure probability may be expressed in an analytical way:

1. If the strength and the stress are normally distributed:

$$P_f = \text{Prob}(R \leq C) = \Phi\left(-\frac{\mu_R - \mu_C}{\sqrt{\sigma_R^2 + \sigma_C^2}}\right) = \Phi\left(-\frac{\mu_R - \mu_C}{\sqrt{(\mu_R CV_R)^2 + (\mu_C CV_C)^2}}\right)$$

where Φ is the cumulative distribution function of the standard normal variable (mean = 0, standard deviation = 1), μ_R, σ_R and CV_R the mean, the standard deviation and the coefficient of variation respectively of the strength variable, and μ_C, σ_C and CV_C the mean, the standard deviation and the coefficient of variation respectively of the stress variable.

Excel ® Formula 2013:

NORM.S.DIST(-(Mu_R - Mu_C)/SQRT(Sigma_R^2+Sigma_C^2);TRUE)

2. If the strength and the stress are lognormally distributed:

$$P_f = \text{Prob}(R \leq C) = \Phi\left(-\frac{\mu_{\ln R} - \mu_{\ln C}}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln C}^2}}\right)$$

where $\mu_{\ln R}$, $\sigma_{\ln R}$ are the mean and the standard deviation respectively of the logarithm of the strength variable and $\mu_{\ln C}$ and $\sigma_{\ln C}$ are the mean and the standard deviation respectively of the logarithm of the stress variable.

Excel ® Formula 2013:

NORM.S.DIST (-(Mu_lnR-Mu_lnC)/SQRT(Sigma_lnR^2+Sigma_lnC^2);TRUE)

The failure probability is very sensitive to the models and parameters of the distributions.

- Example of distribution model impact: the stress C is modeled with a normal distribution of parameters: μ_C = 10 and CV_C = 10 %. The strength R is characterized by an expected value of 15 and a coefficient of variation of 10 %. The following distribution models are assumed for R: normal distribution (which parameters are μ_R = 10 and σ_R = 1.5), lognormal distribution ($\mu_{\ln R}$ = 2.7 and $\sigma_{\ln R}$ = 0.099) and Weibull distribution ($\beta_R \approx 15.64$ and $\eta_R \approx 12.16$). The failure probability varies with a factor 8, according to the results reported in ***Table 20***.

Table 20: Impact of the distribution model in the Stress-Strength method.

R distribution model	Normal	Lognormal	Weibull
P_f	3×10^{-3}	1×10^{-3}	8×10^{-3}

- Example of the distribution parameters impact: the stress C is modeled with a normal distribution of parameters: μ_C = 10 and CV_C = 10 %. The strength R is modeled with a normal distribution of parameters: μ_R = 20 and CV_R whose value varies between 7.5 % and 12.5 %. The failure probability varies with a factor of 10 000 according to the results reported in ***Table 21***.

Table 21: Impact of the distribution parameters in the Stress-Strength method.

CV_R	7.5 %	10 %	12.5 %
P_f	1×10^{-8}	4×10^{-6}	1×10^{-4}

In innovation phase, *i.e.* when no component is in service, a sensitivity study is used to study the distribution parameter impact on the failure probability. A conservative hypothesis is then taken on the most influential parameter.

The parameters of the Stress-Strength model can be adjusted for a component that is already in service. The adjustment consists in comparing the predicted reliability with the real reliability observed among end-users. In case of discrepancy between the 2 probabilities, the hypotheses on the distribution parameters and even maybe on the damaging factors must be revised.

Practical sheet references

[1] *Reliability in Automotive and Mechanical Engineering* - B. Bertsche - Springer - 2008.

[2] *Les systèmes mécatroniques embarqués 2 - analyse des causes de défaillances, modélisation, simulation et optimisation* - Chapter 7 - A. El Hami, P. Pougnet - ISTE Editions - 2015.

[3] *Accéder au juste nécessaire par une expérimentation adaptée* - C. Gomez, G. Perroud – SIA study day « fiabilité expérimentale: essais accélérés et autres techniques pour démontrer un niveau de fiabilité au moindre coût » - 24 October 2000.

In **Practical sheet 6**, the Stress-Strength method is used to assess the reliability based on modeling the loads and the behavior of the component. For designing a test (***Figure 46***), the problem is inverted. The stress distribution $f_C(x,A)$ over a reference period A and the field reliability target over A (=objective failure probability P_f) are known. The method is thus used to characterize the strength $F_R(x)$:

$$P_f = \int_{-\infty}^{+\infty} F_R(x)\ f_C(x,A)dx \rightarrow F_R(x) = \ ...$$

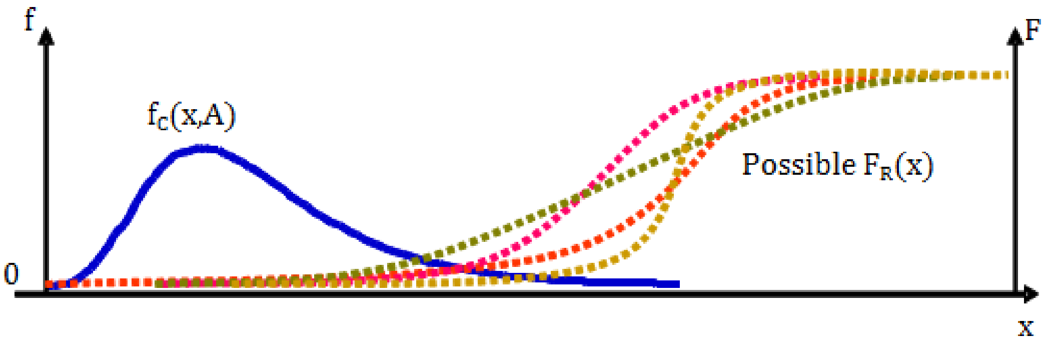


Figure 46: Test design consists in identifying the strength model $F_R(x)$**.**

Performing test to failure is the only way to directly assess all the parameters of the distribution of the strength $F_R(x)$. It is therefore not feasible each time for cost reasons. Another way to proceed, which is less costly, is the time-censored test (or « 0 failure » test). This type of test consists in testing components during a time τ (or a given number of cycles), set in advance. In contrast with the failure test, the objective is not to reach the component failure but to check that there is no failure after τ.

To identify $F_R(x)$, the time-censored test needs experimental feedback on one of the distribution parameters. It is generally on the variability parameter (standard deviation, coefficient of variation, β). Knowing this parameter, denoted θ_{REX}, and the target failure probability P_f over A, the minimum value of the second parameter θ_{mini} is obtained by solving the Stress-Strength inverse problem:

$$P_f = \int_{-\infty}^{+\infty} F_R(x,\theta_{REX},\theta_{mini})\ f_C(x,A)dx \ \rightarrow \theta_{mini} = \ ...$$

Design of the time-censored test

The acceptance criterion of the time-censored test is generally $k = 0$ failure, that is to say no failure is accepted at the end of the test. This criterion allows to express the failure proportion δ_c at τ with a confidence level c:

$$\delta_c = F_R(\tau) = 1 - (1-c)^{1/N}$$

where N is the number of tested components.

The strength parameters are known thanks to experimental feedback and the solution of the inverse Stress-Strength problem. The time-censored test is then designed in order to check that the field reliability target is reached. To do so, the 2 applicable methods are:

1. To define the time duration τ for a given number N of tested components:

$$\tau = F_R^{-1}\left(1-(1-c)^{1/N}, \theta_{REX}, \theta_{mini}\right)$$

2. To define the number N of components to test over a given time duration τ:

$$N = \frac{\ln(1-c)}{\ln(1-F_R(\tau, \theta_{REX}, \theta_{mini}))}$$

If no component fails at the end of the test, the field reliability target is achieved.

Remark: the duration τ is generally chosen in order to have 10 % of the components that fail by this duration:

$$\tau = F_R^{-1}(0.1)$$

Case where there is at least one failure at the end of the test

If failures are observed at the end of the time-censored test ($k>0$), the Bayesian update [1,2] is used to recalculate:

- The confidence level:

$$c = \mathrm{Beta}(F_R(\tau), k+1, N-k+1)$$

where Beta is the Beta cumulative distribution function.

Excel ® Formula 2013: BETA.DIST(FR(τ);k+1;N-k+1;TRUE)

- Or the failure proportion at τ:

$$\delta_c = F_R(\tau) = \mathrm{Beta}^{-1}(c, k+1, N-k+1)$$

where Beta^{-1} is the Beta inverse distribution function.

Excel ® Formula 2013: BETA.INV (c;k+1;N-k+1)

If there are more than 7 failures before the duration τ is reached, then it becomes more interesting to analyze the failures and fit the data with a Weibull distribution (see **Practical sheet 3**). Below 7 failures, it is possible to fit a Weibull distribution provided that the parameter β is known. The value of β can be assumed or an interval can be given using a Bayesian inference technique. In both cases, feedback on the failure mode is required. When the number of tested components is small, the estimation of the failure probability is uncertain. It is recommended to calculate the uncertainty on the estimation of the cumulative distribution function using the Stress-Strength method.

Example

Data:

A time-censored test must be designed to verify the field reliability target of a component. The input data of the Stress-Strength problem are:

- A failure probability target of 10^{-6}.
- A stress expressed in hours and modeled with a log-normal distribution whose parameters are: $\mu_{\ln C} = 5$ et $\sigma_{\ln C} = 0.15$.
- A strength expressed in hours and modeled with a log-normal distribution whose coefficient of variation CV_R is known thanks to the experimental feedback and whose value is 0.10 (that is to say $\sigma_{\ln R} \approx 0.10$).

The acceptance criterion of the test is $k = 0$ failure. A confidence level greater than 70 % on the field reliability target is targeted.

Estimation of the mean value of the strength variable:

The mean value $\mu_{\ln R}$ of the strength variable is obtained by solving the inverse Stress-Strength problem:

$$P_f = \Phi\left(-\frac{\mu_{\ln R}-\mu_{\ln C}}{\sqrt{\sigma^2_{\ln R}+\sigma^2_{\ln C}}}\right) \text{ devient } \Phi^{-1}(10^{-6}) = -\frac{\mu_{\ln R}-5}{\sqrt{0.10^2+0.15^2}}$$

where Φ is the cumulative distribution function of the standard normal variable (mean value = 0 and standard deviation = 1). The mean value $\mu_{\ln R}$ of the strength is 5.86.

Excel ® Formula 2013:

MU_LN_R = -NORM.S.INV(0,000001)*SQRT(0,1^2+0,15^2)+5

Design:

The test duration is set to 310 hours. The number of components to test is therefore 10:

$$N = \frac{\ln(1-0.7)}{\ln(1-F_R(310, 0.10, 5.86))} = 9.8$$

Update after the test:

After the test, 1 component out of the 10 has failed before 310 hours. For the same field reliability target of 10^{-6}, the Bayesian update gives a confidence level of:

$$c = \text{Beta}(F_R(310), 1+1{,}10-1+1) = 33.9\ \%$$

Excel Formula ® 2013:

BETA.DIST(LOGNORM.DIST(310;5,86;0,1;TRUE);1+1;10-1+1;TRUE)

The confidence in the result is too small. To increase it, additional components should be tested. For a level of 70 %, we get that N = 21 for k = 1 (no new failure is accepted among the 11 additional components):

$$c = \text{Beta}(F_R(310), 1+1{,}21-1+1) = 70.6\ \%$$

Practical sheet references

[1] *Méthodes avancées d'analyse des bases de données du retour d'expérience* - A. Lannoy, H. Propaccia - Eyrolles - 1994.

[2] *Fiabilité des équipements et théorie de la décision statistique fréquentielle et bayésienne* - H. Propaccia et L. Piepszownik - Eyrolles - 1992.

Practical sheet 8: Extrapolation of the time-censored tests

Test to failure or failure test is the test that provides the most information about the strength distribution of the component. Time-censored results can be extrapolated to failure test results if the degradation level of the component is measurable, *e.g.* a crack size for crack propagation or a coating thickness for wear.

Method

The method for extrapolating a time-censored test is depicted in ***Figure 47***. The steps are the following:

1. Conduct the time-censored test of a duration (or number of cycles) τ following the **Practical sheet 7**. **The value chosen for τ should respect the condition 2.c below.**
2. Check the 3 following conditions which are needed to extrapolate the time-censored test results:
   a) A relation between the degradation level and the duration (or number of cycles) is needed. For many phenomena such as wear, a linear trajectory model is used. This hypothesis is conservative because the degradation speed of each component is unique and constant. It leads to a standard deviation of degradation that is proportional with service life. If the degradation speed varies with time, the relation between the degradation level and the duration/number of cycles can be simulated with a stochastic process [1]. The standard deviation of the degradation obtained with this approach increases more slowly with time than the trajectory model one.
   b) Failure must occur at the same critical degradation level (or degradation threshold) for all components. For example, a critical crack size propagation, a coating thickness of 0 for wear.
   c) The duration (or number of cycles) τ of the time-censored test must be defined so that a sufficient degradation is generated. The minimum duration is the B10:

   $$\tau = F_R^{-1}(0.1, \theta_{REX}, \theta_{mini})$$

   where F_R^{-1} is the inverse distribution function of the strength whose parameter θ_{REX} is known by feedback. θ_{mini} is obtained by solving the inverse Stress-Strength problem (see **Practical sheet 7**).
3. Evaluate the duration (or number of cycles) to failure τ' with the relation in 2.a and the critical degradation level of 2.b.
4. Apply the methods of the **Practical sheet 3** to the values τ'. The experimental feedback used in 2.c is no longer necessary to define the strength distribution.

Remarks:

- The duration τ is not required to be similar for all the tested components.
- If failure occurs at $\tau^* < \tau$ during the test, then: $\tau' = \tau^*$.

This practical sheet explains how to extrapolate a time-censored test to failure. However, it is also possible to extrapolate the test to a larger duration (for example: a reference period of 15 years or 250 000 km). In this case, the statistical distribution fitted to the extrapolated data no longer characterizes the variability of the duration (or number of cycles) to failure but the variability of the degradation for a given duration (or number of cycles). This approach is used in the comparison method of Test/Field degradations which is more attractive than the Weibull comparison method (see example 1: reliability study of a brake pad).

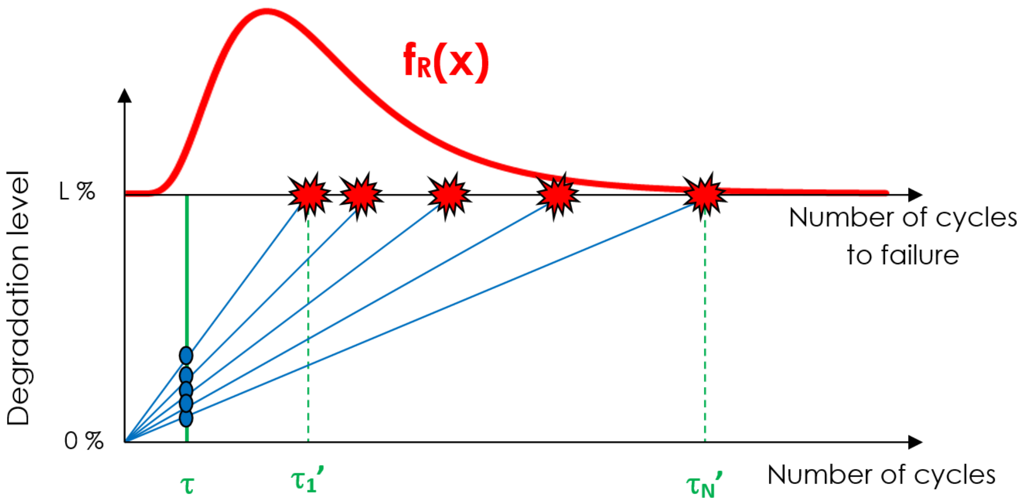


Figure 47: Extrapolation up to failure (degradation level = L) of the test censored at τ and fitting of the distribution of the number of cycles to failure or strength.

Example: wear of a contact of an electronic control

A time-censored test is conducted to check the resistance to wear of a gold layer of initial thickness E_0 = 1 µm on an electronic control contact. The failure defined in condition 2.b of the Method section is a coating thickness of 0.

To check condition 2.c, a 2-parameter Weibull distribution ($\gamma = 0$) is assumed. Its shape parameter β is 2.5 and its scale parameter η is 300 000 cycles. The number of cycles of the time-censored test (B10) is then:

$$\tau > \mathrm{B10} = \mathrm{F_R^{-1}}(0.1, \beta = 2.5, \eta = 300\ 000) = \eta \times [-\ln(1-0.1)]^{\frac{1}{\beta}} = 121\ 953$$

A value of τ =125 000 cycles is considered for this example.
Finally, a linear relation is supposed between the remaining coating thickness E(τ) and the number of cycles (condition 2.a). The number of cycles to failure τ' is then:

$$\tau' = \tau\left(\frac{E_0}{E_0 - E(\tau)}\right)$$

The test results of 8 components are reported in ***Table 22***. The number of cycles to failure τ' is calculated by extrapolation for each component. The **Practical sheet 3** provides methods for fitting the strength distribution to these failure values and this, without using experimental feedback.

Table 22: Results of the time-censored test and extrapolation to failure.

# component	1	2	3	4	5	6	7	8
τ (cycles)	125 000	125 000	125 000	125 000	125 000	125 000	125 000	125 000
E(τ) (µm)	0.65	0.52	0.75	0.67	0.48	0.42	0.33	0.61
τ' (cycles)	357 143	260 417	500 000	378 788	240 385	215 518	186 568	320 513

Practical sheet reference

[1] *Mise au point de modèles prédictifs de fiabilité dans un contexte de dégradation associé à des profils de mission* – Phd Thesis - J. Baussaron - Université d'Angers - 2011.

Practical sheet 9: Reliability test calculation by the Weibayes methodology

The purpose is to provide an aide-mémoire with some of the main relations that are used for calculating censored reliability tests: calculation of the number of parts to be tested, test time, test cost or duration optimization, etc.

Foreward

Introduction

Three cases are presented:

- Censored reliability tests with a single test duration and without failure (elementary hypothesis of the binomial distribution)
- Accelerated censored reliability tests with possible various test durations, but without failure (introduction of the additional Weibull's distribution hypothesis)
- Accelerated censored reliability tests with possible various test durations, and with possible failures (WeiBayes relation; this latter relationship is more general and covers the previous cases)

Notations

The following notations will be used.

R_t	Minimum reliability (e.g. 99%, 99.9%, etc.), associated with a service lifetime **t**, that must be demonstrated after the reliability tests with a confidence level **c**
c	Confidence level (e.g. 80%, 90%, 95%, etc.)
t	"Customer service lifetime" (duration, mileage, number of cycles, etc.), for which the minimum reliability must be demonstrated (e.g. 7 years, 100 000 km, 50 x 10^6 cycles)
t_{max}	"Maximum relevant lifetime", at which point new failure modes that are not representative of the phenomenon under study might emerge (e.g. 300 000 km, etc.)
τ (or τ_i if various tests)	"Test time" (e.g. 1000 h, 15 000 km, 100 000 cycles, etc.)
a (or a_i if various tests)	Acceleration Factor: $a = \tau_{unaccelerated} / \tau_{accelerated}$ (e.g. x 10)
N (or N_i if various tests)	Number of parts tested (to few pieces should be avoided) (Note: if different tests, $N = \sum N_i$)
β	Weibull distribution Shape parameter (e.g. 0.5, 1.0, 2.0, etc.)

Preliminary remarks:

- If the reliability test stress level is the same as the customer service one, then: $a = 1$
- In addition, it is recommended / required to comply with the following condition (the reliability tests must cover at least the customer service lifetime):

$$a.\tau \geq t \quad \Leftrightarrow \quad (a.\tau)/t \geq 1$$

- Moreover, an extended test severity should be avoided in order not to create possible occurrences of new failure modes that are not representative of the problem under study:

  $a \cdot \tau < t_{max} \Leftrightarrow (a \cdot \tau) / t < t_{max} / t$

- The Acceleration factor **a** can be estimated experimentally or using laws such as Arrhenius', Coffin-Manson's, Inverse Power's, Basquin's, etc.

- If a minimum threshold t_0 is necessary before the onset of a failure, then the following customer service life will be analysed:

  $t' = t - t0$.

- Before any mathematical calculation, the engineer will investigate the physics of failure relative to each failure mode and failure mechanism.

Examples

Some examples will be provided in this sheet.
They are for didactic purpose only, and do not represent real examples.

- *Preliminary questions:*
  - *We would like to conduct reliability tests to demonstrate a minimum customer reliability of 99.99% at 100,000 km, with a confidence level of 95%*
  - *In addition, there are risks of wear phenomena beyond 300,000 km, and the appearance of failure modes beyond this mileage that are not representative of the study.*

- *Notations:*
  - *The customer service lifetime is:* t = 100 000 km
  - *The minimum reliability to be demonstrated at 100,000 km is:* $R_{100\,000\,km}$ = 99,99% = 0,9999
  - *The expected confidence level is:* c = 95% = 0,95
  - *The maximum relevant lifetime is:* t_{max} = 300 000 km

Censored reliability tests (single test duration for all tests, no failure at the end of the tests)

A reliability test is considered where all the parts tested are identical and subjected to the same test conditions and duration.

There is no failure at the end of the test.

Key Relations

The minimum reliability **Rt** at a service lifetime **t**, which can be deduced from the failure-free test of **N** components with a confidence level **c**, is:

$$R_t = (1 - c)^{\frac{1}{N}} \qquad (1a)$$

The minimum number **N** of components to be tested in a 0 failure test plan, to demonstrate a minimum Reliability **Rt** at a service lifetime **t**, and with a confidence level **c**, is (this relation is mathematically equivalent to the previous one):

$$N = \frac{ln(1 - c)}{ln(R_t)} \qquad (1b)$$

Remarks

In case of failure(s), the binomial distribution, the Larson nomogram, or the WeiBayes relation (see below) can be used.

The figures below show the values of the minimum reliability "**Rt**", as well as "**1 – Rt**", that can be inferred from failure-free reliability test depending on the number of parts tested and the confidence level.

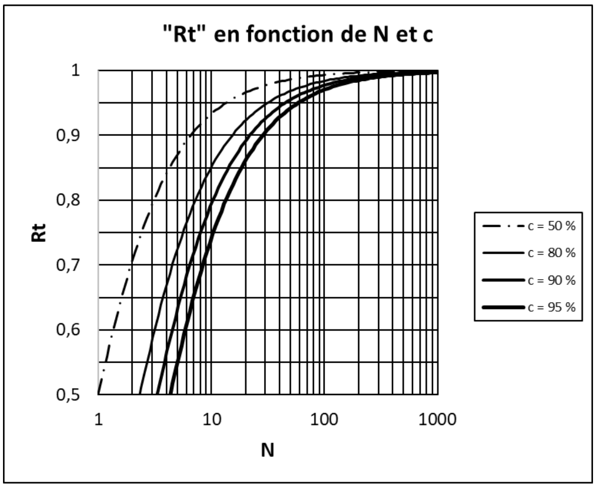


Figure 48: Minimum reliability values from reliability test without failure according the sample size en the confidence level.

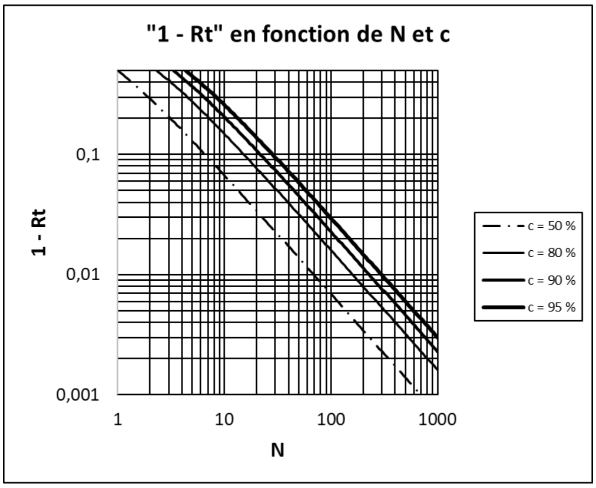


Figure 49: Minimum unreliability values from reliability test without failure according the sample size and the confidence level.

Examples

Questions:

- ***1)*** *With a 0 failure test plan, what would be the minimum number of parts to be tested to prove a minimum Reliability of 99.99% at 100 000 km, with 95% confidence, under identical tests conditions?*

- ***2)*** *For practical reasons, it is not possible to test the number of components that has been calculated under question #1 assumptions.*
  A compromise is defined with the following less ambitious objectives: minimum reliability of 99% at 100,000 km, with a confidence level of 90%.
  What would then be the minimum number of parts to be tested?

Solutions:

- ***1)*** *The minimum number of parts to be tested, under conditions #1, can be calculated using relation (1b):*

  $$\boldsymbol{N} = \frac{\boldsymbol{ln}(1-0{,}95\,)}{\boldsymbol{ln}(0{,}9999)} = \;29\,955{,}8$$

  The value of $\boldsymbol{N}$ *corresponds to the upper value, i.e.:* $\boldsymbol{N} = \mathbf{29\,956}$

  To prove a minimum reliability of 99.99% with 95% confidence, at least 29,956 components need to be tested at the equivalent of 100,000 km service time, and no failure must be observed at the end of the tests.

- ***2)*** *The minimum number of parts to be tested, under the new conditions #2, becomes by application of relation (1b):*

  $$\boldsymbol{N} = \frac{\boldsymbol{ln}(1-0{,}9\,)}{\boldsymbol{ln}(0{,}99)} = \;229{,}1$$

  The value of $\boldsymbol{N}$*, corresponding to the upper value, then becomes:* $\boldsymbol{N} = \mathbf{230}$
  The definition of less ambitious targets has made it possible to significantly reduce the number of parts to be tested. However, the test time remains too long in practice and accelerated tests will be defined.

Accelerated censored reliability tests (various test durations, no failure at the end of the tests)

Reliability tests with possible various conditions are considered.

The different tests have the following characteristics: numbers of parts $\mathbf{N_i}$, durations $\boldsymbol{\tau_i}$ and acceleration factors $\mathbf{a_i}$.

No failure is observed at the end of the tests.

It is also assumed that the probability of failure follows a Weibull distribution with a shape parameter $\boldsymbol{\beta}$.

Key Relations

When $\mathbf{k}$ accelerated tests are carried out and no failure occurs, the minimum reliability $\mathbf{R_t}$ at a lifetime $\mathbf{t}$, which is deduced with a confidence level $\mathbf{c}$, is:

$$\boldsymbol{R_t} \;=\; (\mathbf{1}-\boldsymbol{c})^{\left(\sum_{i=1}^{k} N_i \cdot \left(\frac{a_i\,\cdot\,\tau_i}{t}\right)^{\beta}\right)^{-1}} \;=\; (\mathbf{1}-\boldsymbol{c})^{\left(\frac{1}{\sum_{i=1}^{k} N_i \cdot \left(\frac{a_i\,\cdot\,\tau_i}{t}\right)^{\beta}}\right)} \qquad (2a)$$

The minimum number of components to be tested to demonstrate a minimum reliability $\mathbf{R_t}$, at a lifetime $\mathbf{t}$ and a confidence level $\mathbf{c}$, in a failure-free test, is (this relation is mathematically equivalent to the previous one):

$$\sum_{i=1}^{k} N_i \cdot \left(\frac{(a_i \cdot \tau_i)}{t}\right)^{\beta} = \frac{ln(1-c)}{ln(R_t)} \qquad (2b)$$

Remarks

- The shape parameter **β** value corresponds to a prior knowledge or a conservative estimate.

- If the test conditions and the parts are identical, relations (2a) and (2b) can be written:
  - $R_t = (1-c)^{\frac{1}{N \cdot \left(\frac{(a \cdot \tau)}{t}\right)^{\beta}}}$
  - $N \cdot \left(\frac{(a \cdot \tau)}{t}\right)^{\beta} = \frac{\ln(1-c)}{\ln(R_t)}$

- If $\frac{(a_i \cdot \tau_i)}{t} = 1$, the relations (2a) and (2b) are equivalent to the relations (1a) and (1b).

- Under certain conditions, mathematical calculations could lead to a very small number of parts to be tested, or even to a single part. This would make the test dependent on the part being tested and would be unrepresentative. This is why it is recommended, whatever the case, to test a minimum number of parts (it is sometimes mentioned at least three parts for example).

Examples

Background

- *The objective is to demonstrate a minimum reliability of 99% at 100,000 km ($R_{100,000\,km}$ = 0.99) with a confidence level of 90% (c = 0.9).*
  The maximum lifetime is: t_{max} = 300,000 km
- *Accelerated tests are defined by hardening the mechanical and thermal stresses, without generating new failure modes that would not be representative of the phenomenon studied.*
  Knowledge of similar products leads to an acceleration factor of $a = 10$ (this value is also verified by Basquin and Arrhenius models).

- *It will be assumed that no failure is observed at the end of the tests in the following questions.*

Questions:

- ***1a)** Calculate the minimum number of parts to be tested, when the quantity $(a \cdot \tau)/t$ is equal to **1**, **2** or **3** and the shape parameter of Weibull's distribution β is **0.5**, **1** or **2**.*

- ***1b)** Estimate the test costs if the test cost is proportional to the test bench immobilization, i.e. $N \cdot \tau$ as a first approximation.*
- ***2a)** In this question **2**, it is assumed that the shape parameter of Weibull's distribution is now $\beta = 2$.*
  The accelerated test bench test campaign is planned with 26 parts for 30,000 km.
  After 10,000 km of accelerated testing, the test was interrupted. Not a single part failed.
  What minimum level of reliability could be deduced then with a 90% confidence level?

- ***2b)** During the previous interruption of the test at 10,000 km, 3 parts have been taken for analysis and will no longer be tested, while the other 23 parts will continue the accelerated reliability tests up to 30,000 km.*
  What is then the number of additional parts that must be added (n' = 1, or 2, ...) and the test time necessary to demonstrate the initial reliability objective, with the assumption that no failure occurs at the end of the tests?

Solutions:

- ***1a)*** *The relation (2b) allows to calculate the minimum number of parts to be tested for* $R_{100,000\,km} = 0.99$ *and* $c = 0.9$*. For example:*

$$\beta = 0{,}5 \text{ and } (a\,.\,\tau)/t = 2 \Rightarrow N_{0,5;2}\,.\,(2)^{0,5} = \frac{ln(1-0{,}9)}{ln(0{,}99)} \Rightarrow N_{0,5;2} \geq 162{,}002 \Rightarrow N_{0,5;2} \geq 163$$

The calculations are similar for the other cases and the table below summarizes the results:

N		**β**		
		0,5	***1***	***2***
(a.τ)/t	***1***	*230*	*230*	*230*
	2	*163*	*115*	*58*
	3	*133*	*77*	*26*

Extending the test time reduces the number of tested parts. On the other hand, it is necessary to be vigilant about the risk of occurrence of unrepresentative failure modes and not to exceed the maximum relevant test time.

- ***1b)*** *Since the coefficients* ***a*** *and* t *are known, it is possible to deduce the value of* τ *from* $(a\,.\,\tau)/t$ *(because* $\tau = ((a\,.\,\tau)/t)\,.\,t/a$*), and then deduce* $N\,.\,\tau$*, based on the results of the previous question. The table below summarizes the results:*

(a.τ)/t		=>	***τ (km)***		=>	***N . τ (x 10^6 km)***		**β**		
								0,5	***1***	***2***
	1			***10 000***		***τ (km)***	***10 000***	*2,3*	*2,3*	*2,3*
	2			***20 000***			***20 000***	*3,3*	*2,3*	*1,2*
	3			***30 000***			***30 000***	*4,0*	*2,3*	*0,8*

Unlike the case where $\beta > 1$*, extending the test duration can lead to a bench immobilization cost increase if the parameter* β *value is lower than 1.*
On the other hand, the test cost is independent of the test time when $\beta = 1$*.*

- ***2a)*** *After 10 000 km of accelerated tests,* $\tau 0/t = 10\,000 / 100\,000 = 0.1$
(Note: $(a\,.\,\tau_0)/t \geq 1$ *and the test duration covers the objective (minimum level of customer service life time)*
According to relation (2a), after 10,000 km accelerated tests and no failure of 26 parts, the minimum demonstrated reliability with a 90% confidence level is therefore:
$R_{100\,000\,km} = (1-0{,}9)^{(26\;(10\,.\,0{,}1)^2)^{-1}} = 92\,\%$
This is still insufficient in relation to the objective of minimum reliability to be demonstrated and accelerated tests must be continued.

- ***2b)*** *If* τ' *is the desired test duration corresponding to* n' *additional parts, one writes according to the relation (2b) (knowing that* $R_{100\,000\,km} = 0.99$ *;* $c = 0.9$*;* $\beta = 2$*;* $a = 10$ *; no failure at the end of the tests):*

$$3\,.\left(10\,.\frac{10\,000}{100\,000}\right)^2 + 23\,.\left(10\,.\frac{30\,000}{100\,000}\right)^2 + n'\,.\left(\frac{(a\,.\,\tau')}{t}\right)^2 = \frac{ln(1-0{,}9)}{ln(0{,}99)} = 229{,}1$$

And: $$n'\,.\left(\frac{(a\,.\,\tau')}{t}\right)^2 = 229{,}1 - \left(3\,.\left(10\,.\frac{10\,000}{100\,000}\right)^2 + 23\,.\left(10\,.\frac{30\,000}{100\,000}\right)^2\right) = 19{,}1$$

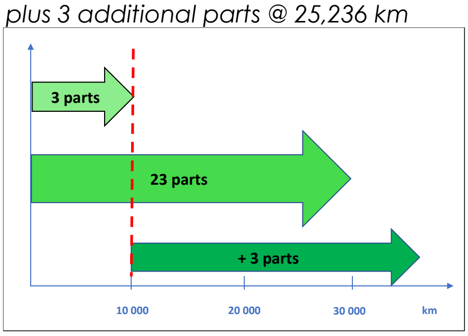
The table below summarizes the results according to the number of additional pieces n'*.*
Note: the condition $t \leq a\,.\,\tau' < t_{max}$ *is equivalent to* $1 \leq (a\,.\,\tau')/t < 3$ *in this case.*

	n'								
	…	**2**	**3**	**4**	**5**	**…**	**19**	**20**	**…**
(a.τ')/t	**…**	**3,09**	**2,52**	**2,19**	**1,95**	**…**	**1,00**	**0,98**	**…**
Decision	$a\,.\,\tau' > t_{max}$	$a\,.\,\tau' > t_{max}$	$t < a\,.\,\tau' < t_{max}$	$t < a\,.\,\tau' < t_{max}$	$t < a\,.\,\tau' < t_{max}$	$t < a\,.\,\tau' < t_{max}$	$t < a\,.\,\tau' < t_{max}$	$a\,.\,\tau' < t$	$a\,.\,\tau' < t$
	NOK	NOK	OK	OK	OK	OK	OK	NOK	NOK
τ' *(km)*	…	*30907*	*25236*	*21855*	*19548*	…	*10028*	*9774*	…
n' . τ' (x 10^4 km)	**…**	**6,18**	**7,57**	**8,74**	**9,77**	**…**	**19,05**	**19,55**	**…**

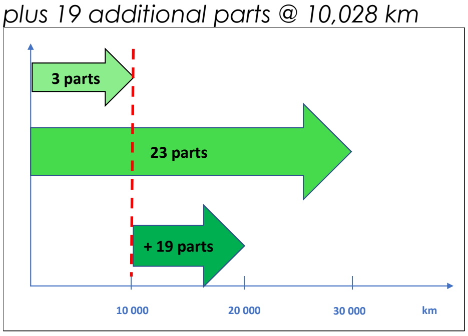
If $n' \leq 2$, then the maximum relevant lifetime would be exceeded. It is NOK, because there is a risk of generating unrepresentative failure modes (and in addition, the number of parts tested would be too low).
If $n' \geq 20$, then the test duration would correspond to a lower value of the customer service lifetime than the objective. It is NOK.

From a cost perspective, the decision depends on the case and the practical constraints. Hereafter, some examples (the number of possibilities is not limited to these options alone).
It is assumed that the cost corresponds to the bench immobilization and is approximately proportional to $n'.\tau'$ (with $\tau' = ((a.\tau')/t).t/a$).

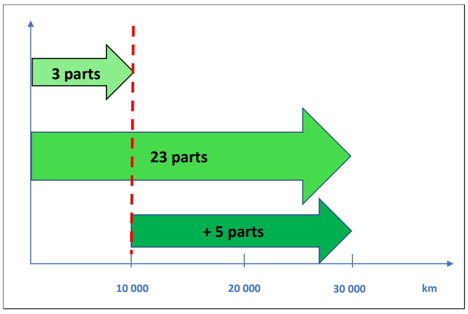
- *If the target is to minimize the bench immobilization cost or the number of parts, the following option would be retained n' = 3 leading to $(a.\tau')/t$ = 2.52, τ' = 25,236 km (cost proportional to $n'.\tau'$ = 7.57 x 10^4 km).*
  This is the following test:
  - *3 parts @ 10,000 km*
  - *23 parts @ 30,000 km (10,000 km + 20,000 km)*
  - *plus 3 additional parts @ 25,236 km*



- *If the target is to free up the bank with the additional parts as quickly as possible, then the following option would be retained $(a.\tau')/t$= 1.00 and n'= 19, τ'= 10,028 km (cost proportional to $n'.\tau'$ = 19.05 x 10^4 km).*
  This is the following test:
  - *3 parts @ 10,000 km*
  - *23 parts @ 30,000 km (10,000 km + 20,000 km)*
  - *plus 19 additional parts @ 10,028 km*



- *If the target is to minimize the handling (the second phase of testing will end simultaneously for all parts), then the following option would be retained $(a.\tau')/t$ = 1.95 and n'= 5, τ'= 19,548 km (cost proportional to $n'.\tau'$ = 9.77 x 10^4 km).*
  This is the following test:
  - *3 parts @ 10,000 km*
  - *23 parts @ 30,000 km (10,000 km + 20,000 km)*
  - *plus 5 additional parts @ 19,548 km (≈ 20,000 km)*



Accelerated censored reliability tests (various test durations, possible failures at the end of the tests)

Reliability tests are considered with various testing conditions

The different tests have the following characteristics: number of parts $\mathbf{N_i}$ and test durations τ_i.

Accelerated tests have a common acceleration factor **a**.

Failures can be recorded at the end of the tests: **x** failures in total.

It is also be assumed that the probability of failure follows a Weibull distribution with a shape parameter $\boldsymbol{\beta}$.

Key Relations

The minimum reliability $\mathbf{R_t}$ at a lifetime $\mathbf{t}$, which is deduced with a confidence level **c,** from reliability tests of $\mathbf{N_i}$ components with possible failure(s) at the end of the tests, is defined by the WeiBayes relation (where $\chi^2_{2x+2\,;\,c}$ is the quantile of the **Chi-Square** distribution with $\mathbf{2x+2}$ degrees of freedom for a probability of $\mathbf{c}$):

$$\boldsymbol{R_t} \;=\; \boldsymbol{exp}\left(-\frac{\boldsymbol{\chi^2_{2x+2\,;\,c}}}{\mathbf{2}\;\sum_{\mathbf{i=1}}^{\mathbf{k}} \boldsymbol{N_i}\;\left(\frac{(\mathbf{a}\,.\boldsymbol{\tau_i})}{\boldsymbol{t}}\right)^{\boldsymbol{\beta}}}\right) \qquad (3a)$$

The minimum number of components to be tested, to demonstrate a minimum reliability $\mathbf{R_t}$ at a lifetime **t** with a confidence level $\mathbf{c}$, during a reliability test with or without failure of $\mathbf{N_i}$ components, is (this relation is mathematically equivalent to the previous one):

$$\sum_{\mathbf{i=1}}^{\mathbf{k}} \boldsymbol{N_i}\;\left(\frac{(\mathbf{a}\,.\,\boldsymbol{\tau_i})}{\boldsymbol{t}}\right)^{\boldsymbol{\beta}} \;=\; -\,\frac{\boldsymbol{\chi^2_{2x+2\,;\,c}}}{2\,.\,\mathbf{ln}(\boldsymbol{R_t})} \qquad (3b)$$

Remarks

If the test conditions and the parts under test are identical, relations (3a) and (3b) can be written:

- $\boldsymbol{R_t} \;=\; \boldsymbol{exp}\left(-\frac{\boldsymbol{\chi^2_{2x+2\,;\,c}}}{\mathbf{2}\,\boldsymbol{N}\,.\left(\frac{(\mathbf{a}\,.\boldsymbol{\tau})}{\boldsymbol{t}}\right)^{\boldsymbol{\beta}}}\right)$
- $\boldsymbol{N}\,.\left(\frac{(\mathbf{a}\,.\boldsymbol{\tau})}{\boldsymbol{t}}\right)^{\boldsymbol{\beta}} \;=\; -\,\frac{\boldsymbol{\chi^2_{2x+2\,;\,c}}}{2\,.\;\mathbf{ln}(\boldsymbol{R_t})}$

- The following table provides some values of **Chi-Square** as a function of the number of failures **x** and the confidence level **c**:

$\chi^2_{2x+2\,;\,c}$		c					
		50%	80%	90%	95%	99%	...
x	0	1,3863	3,2189	4,6052	5,9915	9,2103	...
	1	3,3567	5,9886	7,7794	9,4877	13,2767	...
	2	5,3481	8,5581	10,6446	12,5916	16,8119	...
	3	7,3441	11,0301	13,3616	15,5073	20,0902	...
	4	9,3418	13,4420	15,9872	18,3070	23,2093	...
	5	11,3403	15,8120	18,5493	21,0261	26,2170	...
	6	13,3393	18,1508	21,0641	23,6848	29,1412	...
	7	15,3385	20,4651	23,5418	26,2962	31,9999	...
	8	17,3379	22,7595	25,9894	28,8693	34,8053	...
	9	19,3374	25,0375	28,4120	31,4104	37,5662	...
	10	21,3370	27,3015	30,8133	33,9244	40,2894	...
	...	...	...	...	...	...	...

Excel provides the **Chi-Square** values. For example, the English version:

« =CHISQ.INV (c ; 2.x+2) »

If there is no failure and only a common acceleration factor, the WeiBayes relations (3a) and (3b) are equivalent to the relations (2a) and (2b).

Since: $\frac{\chi^2_{2\,;\,c}}{2} = -\,\boldsymbol{ln}(\mathbf{1} - \boldsymbol{c})$

Although the WeiBayes relations are slightly more complex to compute due to the introduction of the Chi-Square function, they are more general and allow calculations in case of failure(s).

This is very important in practice, as well as from an economic point of view. Indeed, it makes it possible to:

- use the information from the tests already performed in case of failure(s), and consequently reduce the number of additional tests to be tested (and therefore the costs of testing)
- estimate and optimize the number of additional tests that might be required in case of failure(s) when planning the reliability tests.

Examples

Questions:

- ***1)*** *If there is a single acceleration factor common to all accelerated tests and there is no failure at the end of the tests, verify that the WeiBayes relations (3a) and (3b) are equivalent to the relations (2a) and (2b) when the confidence level* $\boldsymbol{c}$ *is 50%, 80%, 90% or 95%.*

- ***2)*** *The objective is to demonstrate a minimum reliability of 99% at 100,000 km (*$R_{100,000\,km} = 0.99$*) and a confidence level of 90% (*$\boldsymbol{c} = 0.9$*).*
  The maximum relevant lifetime is $t_{max} = 300,000$ *km.*
  The shape parameter of the Weibull's distribution is $\beta = 2$*.*
  Accelerated reliability tests are defined with an acceleration factor $\boldsymbol{a} = 10$*.*
  26 parts are subjected to accelerated tests lasting 30,000 km.
  - ***2a)*** *Calculate the minimum reliability with a 90% confidence level using the WeiBayes relation if there is no failure, or if there is one failure at the end of the reliability tests.*

- ***2b)*** *It is assumed that there has been one failure in the reliability tests defined above. Calculate the number of additional parts* n'' *and the test duration* τ'' *necessary to demonstrate the initial reliability objective, assuming that no further failures will occur during the additional tests.*

<u>Solutions</u>:

- ***1)*** *Indeed, in the absence of failure and with a single acceleration factor, the WeiBayes relations (3a) and (3b) are equivalent to the relations (2a) and (2b) if:*

  $$\frac{\chi^2_{2\,;\,c}}{2} = -\,\boldsymbol{ln}(1-c)$$

  The following table checks this property for some confidence levels:

	c				
	50%	*80%*	*90%*	*95%*	*99%*
$\frac{\chi^2_{2\,;\,c}}{2}$	*0,6931*	*1,6094*	*2,3026*	*2,9957*	*4,6052*
$-\,\boldsymbol{ln}(1-c)$	*0,6931*	*1,6094*	*2,3026*	*2,9957*	*4,6052*

- ***2a)*** *If there is no failure:* $\mathbf{x} = 0$ => $2\,\mathbf{x} + 2 = 2$. *And:.* $\chi^2_{2\,;\,0,9} = 4{,}6052$
  So the minimum reliability at 100,000 km with a 90% confidence level can be deduced thanks to the relation (3a):

  $$R_t = exp\left(-\frac{\chi^2_{2\,;\,0,9}}{2\,.\,(26\ (10\,.\,0{,}3)^2)}\right) = 99{,}0\ \%$$

  If there is one failure: $x = 1$ => $2\,x + 2 = 4$. *And:.* $\chi^2_{4\,;\,0,9} = 7{,}7794$
  So the minimum reliability level at 100,000 km with a 90% confidence level becomes:

  $$R_t = exp\left(-\frac{\chi^2_{4\,;\,0,9}}{2\,.\,(26\ (10\,.\,0{,}3)^2)}\right) = 98{,}4\ \%$$

- ***2b)*** *If* τ'' *is the test duration corresponding to the* n'' *additional parts, one writes according to the relation (3b), knowing that there is no other failure at the end of the tests and that* $R_{100{,}000\,km} = 0.99$*;* $c = 0.9$*;* $\beta = 2$*;* $a = 10$*:*

  $$26\,.\left(10\,.\frac{30\ 000}{100\ 000}\right)^2 + n''\,.\left(\frac{(a\,.\tau'')}{t}\right)^2 = -\frac{\chi^2_{4\,;\,0,9}}{2\,.\,\boldsymbol{ln}(0{,}99)} = 387{,}0$$

  The table below summarizes the results (knowing that in this case, the $t \leq a\,.\,\tau'' < t_{max}$ *condition is equivalent to* $1 \leq (a.\tau'')/t < 3$*):*

	n''						
	…	**17**	**18**	**…**	**153**	**154**	**…**
(a.τ'')/t	**…**	**3,000**	**2,916**	**…**	**1,000**	**0,997**	**…**
Decision	$a\,.\,\tau'' > t_{max}$	$a\,.\,\tau'' > t_{max}$	$t < a\,.\,\tau'' < t_{max}$	$t < a\,.\,\tau'' < t_{max}$	$t < a\,.\,\tau'' < t_{max}$	$a\,.\,\tau'' < t$	$a\,.\,\tau'' < t$
	NOK	NOK	OK	OK	OK	NOK	NOK
τ'' *(km)*	…	*30002*	*29157*	…	*10001*	*9968*	…
n'' . τ'' (x 10^4 km)	**…**	**51,00**	**52,48**	**…**	**153,01**	**153,51**	**…**

  If $n'' \leq 17$*, then the maximum relevant lifetime would be exceeded: NOK.*
  If $n'' \geq 154$*, then the customer service lifetime objective would not be fulfilled: NOK.*

  From this table, several reliability testing strategies might be defined:

  - *If it was desired to minimize the number of additional parts or the bench immobilization (proportional to* $n''.\,\tau''$ *as a first approximation, with* $\tau'' = ((a.\tau'')/t)\,.\,t\,/\,a$*), then the option* n'' *= 18 would be retained: the additional test of the 18 parts would be planned over a distance of 29,157 km.*
  - *The student will also have calculated that instead of a sequential approach (testing 26 parts over 30,000 km, and then 18 parts over 29,157 km in case of one failure), another strategy might also be defined under the assumption that there is a risk of one failure. 44 parts would be tested*

up to 22,819 km. At 22,819 km, is there was no failure, the test would be finished; if there was one failure, the test of the 43 remaining parts would continue up to 29,658 km (≈ 30 000 km).

- *The first strategy would be longer in terms of total test time, whether there is a failure or not. On the other hand, it would be more cost effective (number of parts, bench immobilization) in case of no failure, but almost the same as the second strategy in case of one failure.*
- *The choice of a scenario will therefore be made according to the constraints of the project: cost-effectiveness, depending for example on the parts price or availability, tests costs, or time-based with an imperative deadline. It is linked to the specificity of each project. Moreover, the number of scenarii is not limited to these cases alone.*
- *It should also be noted that this type of analysis, as well as the conclusions drawn from it, depend on the value of* β *and the assumption of the number of possible failures that might occur.*

Practical sheet references

[1] *Risk Evaluation Network* –Continental – Dr. R. Schubert, C. Niggel - SIA, 2021.

[2] *Quality Management in the Automotive Industry – Reliability Assurance of Car Manufacturers and Suppliers - Volume 3, part 2 – Verband der Automobilindustrie e.V.* - VDA, German Association of the Automotive Industry – English edition – 2018.

[3] http://www.engineeredsoftware.com/nasa/rt_bayesian.htm

[4] https://www.redalyc.org/journal/496/49645986007/html/#redalyc_49645986007_ref10

Handbooks for estimating the reliability of electronic components

The predictive reliability study of an electronic component is generally limited to the useful life period phase where the failure rate $\lambda(t)$ is constant (***Figure 50***). The early period is not considered because burn-in operations when necessary, allow to sort out components with defects (HASS tests see ***Practical sheet 12***). The wear-out period is not taken into account because the intended use of the electronic system is generally lower to the wear-out of the component.

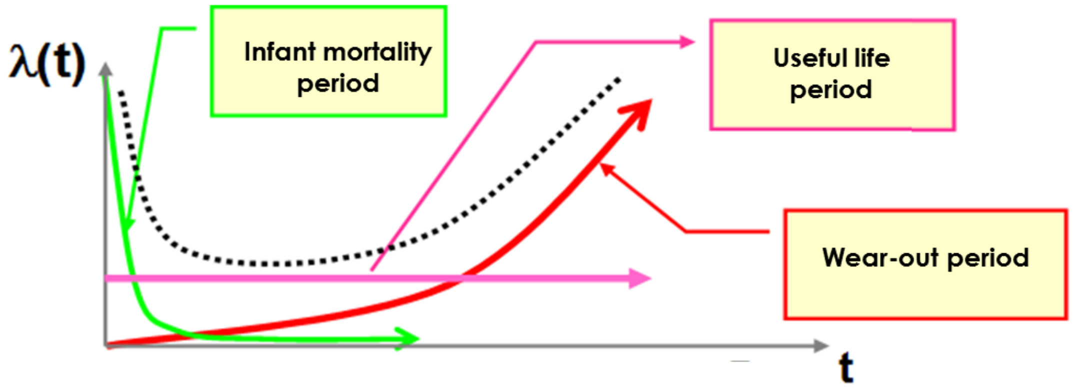


Figure 50: Relation between the life phases of electric components and the failure rate.

Handbooks providing the Predictive reliability in the service life phase have been built based on feedback on failures and maintenance of electronic components. For example: FIDES (UTE C80-811, 2009), MIL-HDBK-217 (1995 for version F Notice2), IEC62380:2004 (UTE C80-810, 2005, formerly RDF), IEC 61709:2017 (merger in 2017 of 61709:2011 and IEC62380:2004), HDBK-217Plus (2015), SN29500 (2004-2011). The failure rates indicated by these handbooks are generally provided with a 50% confidence level and expressed using a generic formula for each component type.

Some handbooks use a multiplicative model (MIL-HDBK-217, IEC61709 and SN29500).

Others involves an additive model (FIDES, HDBK-217Plus, IEC62380) of elementary failure rates, representative of the stresses in operation (temperature, mechanical, humidity, overload ...):

$$\lambda = \lambda_{\text{thermal}} + \lambda_{\text{mechanical}} + \lambda_{\text{overload}} + \cdots$$

Some handbooks as FIDES or HDBK-217Plus also involve parameters reflecting the component manufacturing technical quality or the control of the development / manufacturing / maintenance process of equipment containing the component.

The distribution of the failure rate over the different component failure modes is sometimes not described in certain handbooks. In this case the MIL-HDBK-338B standard or Annex A of IEC 61709 can be used in addition. This distribution is important because a system level effect is often specific to the failure mode (SC, OC, drift, etc.). It will also be necessary for compliance calculations of the various ISO26262 metrics.

The NPRD 2016 (Non Electronic) can also supplement the electronic handbooks collections for electrical module components (e.g. motor).

The handbooks are compared in ***Table 23*** to ***Table 26*** according to the physical stresses and the types of mission profiles considered. FIDES is the most complete handbook for stresses. For mission profiles, the MIL-HDBK-217 only considers a single active phase, the RIAC-HDBK-217 considers a single active phase and a single passive phase while the FIDES and IEC62380 consider all service phases.

Table 23: General handbook comparizon.

	FIDES	MIL-HDBK-217F	HDBK-217Plus(1)	IEC62380	IEC61709	SN29500
References / field data included	Military & Aeronautics	Military	Military	Telecom.& Railway	no(2)	Industrial
Distribution of failure mode	No	no	no(3)	X	Annex A	no
Last update	2009	1995	2015	2004	2017	2004
Handbook maintenability	High	Obsolete	Medium /High	Stopped	High	Medium

(1) 217Plus is an evolution of MIL-HDBK-217, managed by DoD-RIAC then by Quantérion (https://www.quanterion.com/).
(2) IEC61709 gives recommendations for establishing component reliability database (Annex G) but not λ_0.
(3) distribution by categories of causes of failure (Annex A – §Component Reliability Models / Model Form).

Table 24: Model principles and mathematical formula.

Handbooks	Principles	Model – Formula type
FIDES	Physical model of failure mechanisms (1)	$\lambda = (\Sigma\lambda_{physical})(\Pi_{Process})$ $\lambda_{physical} = [\lambda_{0T}\Pi_T + \lambda_{0TCy}\Pi_{TCy} + \lambda_{0M}\Pi_M \ldots].\Pi_{induced}$
MIL-HDBK-217	Multiplicative empirical model	$\lambda = \lambda_b.\Pi_T.\Pi_E.\Pi_V.\Pi_P.\Pi_S.\Pi_Q$
HDBK-217Plus	Statistical model by type of failure mechanism (1)	$\lambda = (\Sigma\lambda_{méca\ de\ def})(\Pi_{Part\_Process})^{(2)(3)}$ $\lambda_{méca\ de\ def} = [\lambda_o\Pi_o + \lambda_E\Pi_E + \lambda_c\Pi_c + \lambda_I + \lambda_r\Pi_r]$
IEC 62380	Statistical model by type of failure mechanism	$\lambda = \lambda_{conducteur} + \lambda_{boitier} + \lambda_{surcharge}$
IEC61709	Multiplicative empirical model	$\lambda = \lambda_{ref}.\Pi_T.\Pi_E.\Pi_U.\Pi_I.\Pi_{ES}.\Pi_{freq\_manoeuvre}$
SN29500	Multiplicative empirical model	$\lambda = \lambda_{ref}.\Pi_T.\Pi_U.\Pi_I.\Pi_S.\Pi_{load}$

(1) combination of additive and multiplicative model,
(2) 217Plus handbook also offers a calculation of a lambda at the system level by taking into account in addition to the $\pi_{Part\_Process}$ factor, factors linked to the design and production processes, to the quality system, as well as the proper consideration of issues such as wear or software quality [2][3],
(3) if sufficient empirical data is available on the new design, the 217Plus handbook also offers an estimate of reliability using a Bayesian approach on the combined basis of the predicted initial data and the first available empirical data (field data or from tests).

Table 25: Handbook comparison according to the constraints taken into account.

Constraints	FIDES	MIL-HDBK-217	HDBK-217Plus	IEC62380	IEC61709	SN29500
Thermal	$\pi_{Thermal}$	π_T	π_o	X	π_T	π_T
Thermal cycle	π_{TCy}		π_c	X		
Mechanical or operation	$\pi_{Mechanical}$	π_E	π_E (1)		$\pi_{operation\_freq}$ π_E (2)	
Thermo-chemical	π_{RH}		π_E			
Chemical	$\pi_{Chemical}$					
Load rate V,I,P	π_{Elec}	π_V, π_P, π_S	π_o	X	π_U, π_I, π_{ES}	π_U, π_I, π_S π_{Load}
Overload	$\pi_{induced}$(3)		π_i	X		
Process	π_{dev}, π_{prod}	π_Q	$\pi_{Part\_Process}$			

(1)take into account at system level but not at component level (factor $\pi_{wearout}$),
(2) partially included in an environmental factor and a 3 levels classification,
(3) the overload factor is calculated for each mission profile phase.

Table 26: Comparison of handbooks regarding the types of mission profiles.

Type de phase	FIDES	MIL-HDBK-217	HDBK-217Plus	IEC62380	IEC61709	SN29500
On	X	X	X	X	X	X
Off	X		X	X	X	X
Multiples	X		X(1)	X	X(2)	

(1) Limited to On/Off cycling frequency,
(2) Details described in Annex D of the standard.

ISO 26262

ISO 26262 standard is a risk management global approach for road vehicles. It aims at ensuring functional safety throughout the life cycle of electrical and electronic systems at the hardware and software levels. It consists in 9 normative parts and a manual as shown in ***Figure 51***.

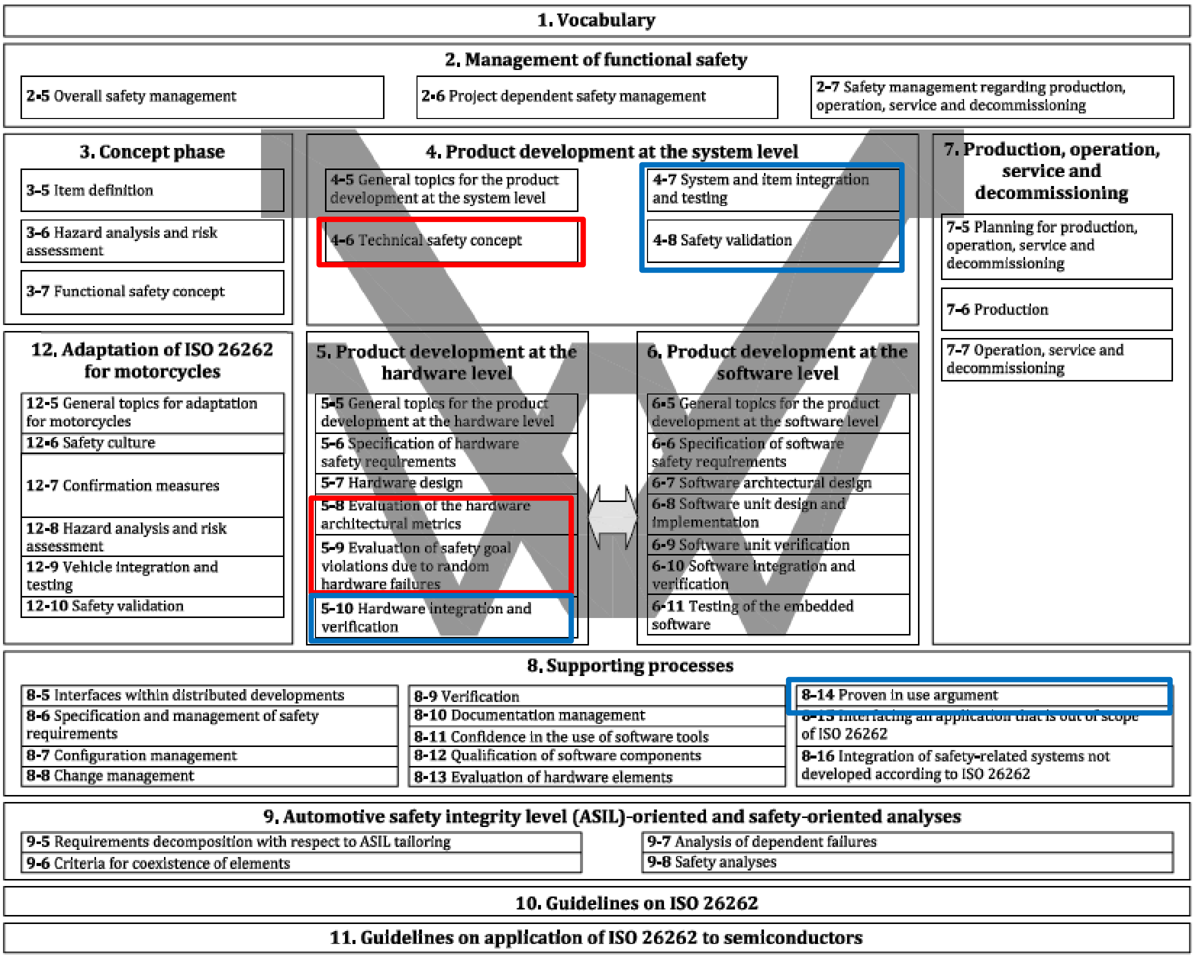


Figure 51: Overview of ISO 26262.

ISO 26262 standard provides requirements and quantitative recommendations (called metrics) and qualitative recommendations in order to control risks.
The chapters of the standard relating to this sheet are framed in red, those relating to the Validation and Safety sheet in blue.

Each hazard event is quoted by an ASIL level (Automotive Safety Integrity Level, ***Table 27***): QM (Quality Management), A, B, C or D, D being the most critical level. This rating is the product of the severity (S1 = light and moderate injuries to S3 = life-threatening to fatal injuries), the exposure (E1 = very low probability to E4 = high probability) and the controllability (C1 = simply controllable to C3 =difficult to control to uncontrollable).

Severity class	Exposure class	Controllability class		
		C1	C2	C3
S1	E1	QM	QM	QM
	E2	QM	QM	QM
	E3	QM	QM	A
	E4	QM	A	B
S2	E1	QM	QM	QM
	E2	QM	QM	A
	E3	QM	A	B
	E4	A	B	C
S3	E1	QM	QM	A[a]
	E2	QM	A	B
	E3	A	B	C
	E4	B	C	D

Table 27: ASIL determination (part of ISO 26262-3).

Requirements and recommendations of ISO 26262 are defined regarding ASIL levels. Examples are given below.

Quantitative requirements:

- Part 5, Section 9.4.2.1: the reliability targets for a random hardware failure.

Table 28: Possible source for the derivation of the random hardware failure target values ISO26262-5 - Table 6.

ASIL	Random hardware failure target values
D	$<10^{-8}$ h^{-1}
C	$<10^{-7}$ h^{-1}
B	$<10^{-7}$ h^{-1}
NOTE The quantitative target values described in this table can be tailored as specified in 4.2 to fit specific uses of the item (e.g. if the item is able to violate the safety goal for durations longer than the typical use of a passenger car).	

- Part 8, Section 14.4.5.2.4: For a proven in use status to be obtained by the candidate, its evaluation period shall demonstrate compliance with each safety goal that can be violated by the candidate in accordance with ISO26262-8 - Table 6 with a single-sided lower confidence level of 70 % (using a chi-square distribution).

Table 29: Limits for observable incident rate ISO26262-8 – Table 6.

ASIL	Observable incident rate
D	$<10^{-9}$/h
C	$<10^{-8}$/h
B	$<10^{-8}$/h
A	$<10^{-7}$/h

• In case of no observable incident, a minimal service period is required:

Table 30: Limits for observable incident rate ISO26262-8 – Table 6.

ASIL	Minimum evaluation period without observable incident
D	$1,2 \times 10^9$ h
C	$1,2 \times 10^8$ h
B	$1,2 \times 10^8$ h
A	$1,2 \times 10^7$ h

Qualitative recommendation:

- Part 5, Section 7.4.1.6: To reduce the failure risk due to high complexity, the hardware architectural design must exhibit the 3 following properties using the principles of Table 1: modularity, adequate level of granularity, simplicity.

Table 31: Properties of modular hardware design - ISO26262-5 - Table 1.

	Properties	ASIL			
		A	B	C	D
1	Hierarchical design	+	+	+	+
2	Precisely defined interfaces of safety-related hardware components	++	++	++	++
3	Avoidance of unnecessary complexity of interfaces	+	+	+	+
4	Avoidance of unnecessary complexity of hardware components	+	+	+	+
5	Maintainability (service)	+	+	++	++
6	Testability[a]	+	+	++	++

[a] Testability includes testability during development, production, service and operation.

- Other qualitative recommendations are defined for the different stages of HW design (safety analysis or design verification, etc.).

Remark: failure rates obtained with reliability handbooks are used to meet the ISO 26262 requirements. For other usages than the ones described in the standard, it is necessary to resort to more specific validations.

Practical sheet references

[1] Road Vehicle – Functional Safety - Part 1: Vocabulary (ISO26262 – Part 1), 2018.

[2] Road Vehicle – Functional Safety - Part 3: Concept Phase (ISO26262 – Part 3), 2018.

[3] Road Vehicle – Functional Safety - Part 5: Product development at the hardware level (ISO26262 – Part 5), 2018.

[4] Road Vehicle – Functional Safety - Part 8: Supporting processes (ISO26262 – Part 8), 2018.

[5] *An Introduction to the RIAC 217PlusTM Component Failure Rate Models", Journal of the Reliability Information Analysis Center* – 2007.

[6] *An Overview of the 217PlusTM System Reliability Assessment Methodology* - Journal of the Reliability Information Analysis Center – 2006.

Methods for validating safety-related reliability objectives

As recalled in the ***Practical sheet 10***, the reliability objectives related to safety concern mainly random failures of electrical/electronic components.

They can be validated in three different ways:

1. **based on reliability handbooks:**

The approach is described in the ***Practical sheet 10***. It is essential that the specific conditions of the use case are considered and satisfactorily covered by the handbook (types and levels of constraints, mission profile...).

2. **based on supplier reliability database:**

Component suppliers assess the random failure rate of their components based either on:

- the field experience resulting from customer incidents and claims. Limitation: not all ground incidents are referred to OEMs leading to bias in the failure rate estimation.
- the results of experimental qualification plans using accelerated temperature tests (most often HTOL - High Temperature Operating Life) conducted as part of the AECQ – Automotive Electronic Council Quality qualifications. Robust estimation can be achieved by a combination of tests performed regularly.
- the basis of reliability handbook.

3. **based on the "Proven in Use":**

The "Proven in Use" is an alternative accepted by the ISO26262 standard to demonstrate a management of safety risks for situations of reuse of existing elements already in production (HW, SW, System, Function...). The "Proven in Use" can be used for products with a high degree of commonality (mission profile, HW, SW...).

The necessary conditions for building a "Proven in use" *demonstration report* are:

- The availability and relevance of field feedback data (completeness, quality of failure analyses, duration of field observations...). Random and systemic failures will be the focus,
- A minimum quantity of similar products in the field,
- The observation time of returns must be longer than the annual duration of the future project (sufficient observation window),
- The list of changes between the observed product and the product envisaged on the new project and a traceability of these changes over time in order to sequence the differences between the different versions,
- Differences in the conditions of use (functionality, mission profile...).

From these observations, an estimated failure rate can be calculated using a Khi2 law with a confidence level of 70%. The ISO26262 standard defines a minimum observation time based on the level of ASIL (see ***Practical sheet 10*** and ***Table 30).***

The assessed failure rate is the random failure rate (constant value), a one-time drift (quality crisis) resolved since must not be taken into account.

In the constant fault area, we only have information during the warranty period. If we demonstrate that wear failures occur well beyond the life (10/15 years) then the information between 0-3 years is relevant. Returns between 3 and 10/15 years will be similar.

The key point is the sampling level between all returns and received parts.

Validation of reliability objectives is also based on:

- The effectiveness of the security mechanisms implemented to prevent failures from leading, on their own, directly to a violation of a security objective (Diagnostic Coverage Part 5 § 9.4.2.4-d),
- The exposure duration in case of "multiple-points fault" (Exposure Duration - Part 5 § 9.4.2.4-e).

Qualitative recommendation

If we consider reliability in the broad sense (beyond wear-type failures: over-stress...), the different types of tests recommended by the standard are as follows:

- Conventional tests:
    - Environmental, mechanical endurance...
- Extreme tests in cases with high quality requirements (especially required for ASIL C and D):
    - "Expanded functional test": Identify extreme scenarios (corner point, outside specification...),
    - «Statistical test»: Define a test based on the distribution of stresses by stress level (gaussian...),
    - "Worst case test" means testing at the limit of the defined specifications,
    - "Over limit test": Robustness test (beyond limits...) (see ***Practical sheet 12***).

Validation tests will have to be carried out at the different levels of system integration with increasing requirements regarding ASIL level:

- HW Validation Tests: Part 5 §10-4-6 – Table 12

Table 32: Hardware integration tests to verify durability, robustness and High stress operability – ISO26262-5 – Table 12.

	Methods	ASIL			
		A	B	C	D
1a	Environmental testing with basic functional verification[a]	++	++	++	++
1b	Expanded functional test[b]	o	+	+	++
1c	Statistical test[c]	o	o	+	++
1d	Worst case test[d]	o	o	o	+
1e	Over limit test[e]	+	+	+	+
1f	Mechanical test[f]	++	++	++	++
1g	Accelerated life test[g]	+	+	++	++
1h	Mechanical Endurance test[h]	++	++	++	++
1i	EMC and ESD test[i]	++	++	++	++
1j	Chemical test[j]	++	++	++	++

- the HW and SW integration validation tests: Part 4 §7-4-2 Table 4,5,6,7,8,
- tests for system and vehicle integration, such as:
    - verification of the proper functioning of the safety mechanisms (performance level, precision, timing): Part 4 §7.4.3 Table 10, §7.4.4 Table 14,
        - Failure injection tests, field tests, in-time tests under real conditions...

- o the robustness verification at vehicle level: Part 4 §7.4.3 Table 12, §7.4.4 Table 16,
    - stress tests, resource use tests, time-to-life tests...

It is necessary to ensure that the safety on demand mechanisms are operational during or at the end of the endurance load cycle and to ensure that no undesired events occur during endurance tests.

Validation method in new areas: connected systems, autonomous vehicles...

1. Context:

With the development of connected systems and ADAS (Advanced Driver Assistance Systems), the number of advanced features implemented in vehicles or even in vehicles and their functional environments (infrastructure, cloud...) is increasing sharply.

Achieving an acceptable level of security requires avoiding any unreasonable risk caused by each hazard associated with the intended functionality and its implementation. To this end, in addition to the hazards due to functional failures and covered by ISO26262, it is also necessary to cover those due to deficiencies in specifications or limitations in performance.

This is why the scope of the ISO26262 standard (malfunctioning behavior in E/E system) has been supplemented by that of standard ISO 21448 – **S**afety **O**f **T**he **I**ntended **F**unctionality (SOTIF) tackling this new area of risk with very miscellaneous:

- sensitivity of a sensor not adapted to certain use cases,
- unspecified use case,
- ergonomics of use not adapted to the user …

2. Challenges:

Hedging these risks requires a different approach than that usually used.

ISO26262 risk coverage is generally ensured by inductive (FMEA) and deductive (Fault Tree) type analyzes based on the failure modes of components that are generally known and in limited number.

The new area of SOTIF risks is characterized by much more numerous, diverse and often unknown causes. It will be covered by an iterative exploration of the functional domain to which the device will be subject. The analytical exploration mode may be inductive and/or deductive, and consist of:

- Exploration by simulations,
- Exploration through laboratory tests, vehicle rolling,
- Exploration on customer fleets in “silent” mode, etc.

This exploration will make it possible to trap unspecified use cases, performance limitations and other special cases leading to unacceptable risks; in order to define the measures necessary to control these risks and to gradually expand the area covered.

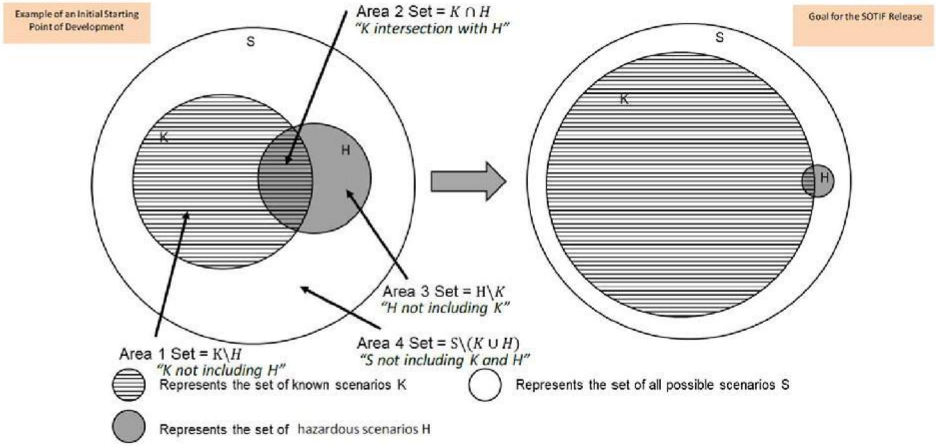


Figure 52: Evolution of the scenario categories resulting from ISO21448 activities – Fig. 7 (extract).

3. Validation Principles:

Validation must demonstrate that the identified risk mitigation actions have been carried out:

- considering additional specifications,
- considering additional use cases,
- improvement in the performance of certain organs,
- restriction of the area of use,
- improvement in user ergonomics (IHM),
- improvement of user training to avoid misuse,
- …

But also, that the development activities (analysis, design, V&V) were sufficiently robust to ensure a level of confidence adapted to the operation of the system in real use conditions.

This leads to an accepted level of confidence regarding a minimized residual risk, without unacceptable risk for vehicle occupants and road users.

Practical sheet references

[1] Road Vehicle – Functional Safety - Part 4: Product development at the system level (ISO26262 – Part 4), 2018.

[2] Road Vehicle – Functional Safety – Part 5: Product development at the hardware level (ISO26262 – Part 5), 2018.

[3] Road Vehicles – Safety of the Intended functionality (ISO 21448), June 2022.

Robustness Test principle

Robustness test is a qualitative test whose objective is to evaluate the robustness of the product by exploring its operation beyond its specifications and discover its weaknesses with respect to its constraints related to its use profile (fuse point).

These tests make it possible to highlight intrinsic weaknesses of the system (design weakness or insufficient design margins between the robustness of a system and these limits of uses). These design margins are evaluated through the operating limit of the product (limit where the system stops working reversibly) and the destruction limit of the product (irreversible limit where the system stops working permanently). Specific margins are evaluated for each identified constraint (Temperature…) one by one at the first time. In a second step, the test can be performed by cumulating the applied stresses beyond their specification.

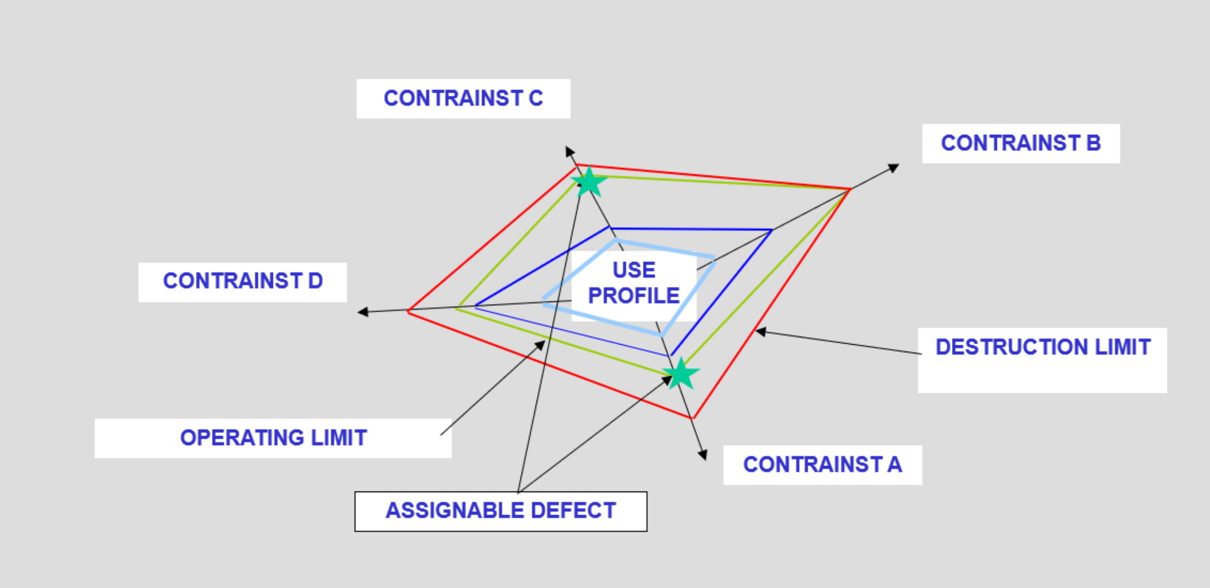


Figure 53: Functional and destruction limits definition.

The principle of this test is to increase the environmental or operating stress gradually to values above the specified values up to the limits of operation or destruction. The robustness test will therefore push the product to its limits, if possible until its failure. The success of aggravated tests lies in the discovery of defects (weak point identified against a stress). So you have to agree to generate failure on the product.

These tests are not intended to estimate product reliability. Robustness testing is a dynamic process to build the reliability of a product. It is not limited to detection of operating and destruction boundaries. Robustness testing pushes the boundaries of the product through a dichotomous approach to design improvement by identifying and then eliminating weaknesses.

This type of test is therefore a design aid tool whose objective is to highlight and correct design weaknesses. It is recommended to deploy them as early as possible in the development phase (proto A). It does not replace validation tests, it is a complement to increase the detection spectrum on all failures.

<u>Point of attention:</u> all the defects precipitated during robustness tests are not representative of the defect's observable in customers (failure mode related to failures outside the conditions of use).

There is no official standard for robustness testing but there are guides defined by some companies: Airbus, Embraer, GM, Case New Holland, GE…

Relevance of the robustness test

The robustness test is of multiple interest:

- Increase the maturity of the design by revealing the systemic weaknesses of the system (early failure...) and evaluating the margins of operation and destruction.
- Improve the robustness of a system by reducing its sensitivity to specifications overruns or process drift that can lead to system failures during its life cycle. The more robust a system is, the more insensitive it is to the temporary exceeding of its specification of use (Less return from the field!).

These robustness tests therefore make it possible to detect design weaknesses as early as the product development phase (proto A) as well as to assess the destruction and operating limits. The duration of these tests (a few days) is small compared to endurance and repetition tests, this is of great interest.

However, these tests do not take into account the mechanisms of slow failures mode (corrosions, migrations, brazed joints, etc.) and thus do not allow the reliability of the system to be assessed.

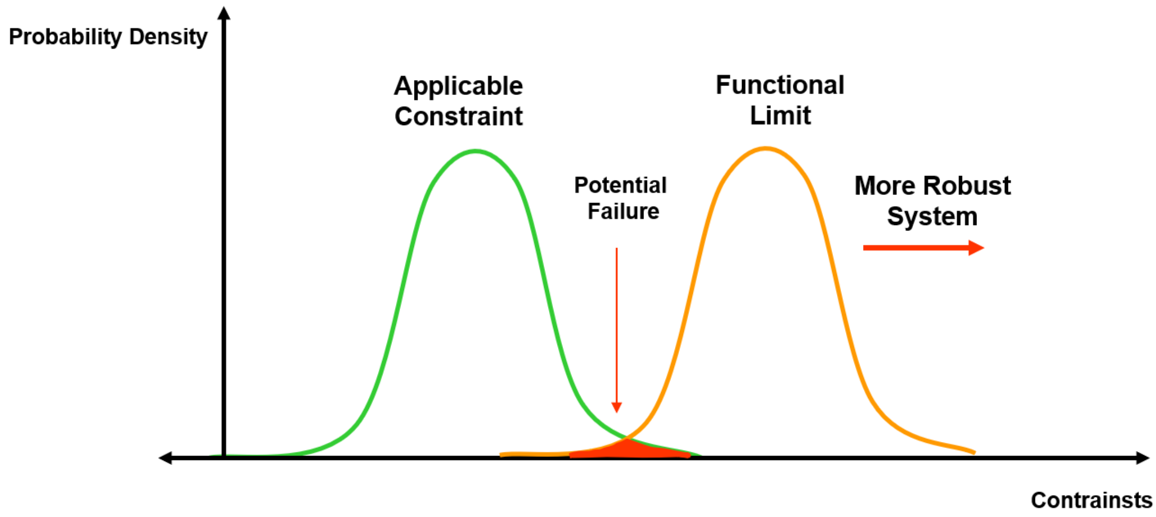


Figure 54: Reduction of Potential Failure.

Robustness tests directly address 3 types of operational failures:

- **Lack of maturity in manufacturing**
    - Connection problem, bad soldering on electronic boards, Assembly problem (screwing…)

  - Components damaged by improper handling
- **Component defect**
  - Component out of supplier specification
  - Packaging issue
  - Contamination generating early failure
  - Poor Sealing….
- **Design errors**
  - Design error (thermal, mechanical, electrical) with respect to the constraints of the mission profile
  - Poor mechanical design
  - Poor technology / application …

They are particularly relevant for systems incorporating high innovation level to compensate for the lack of lesson learn and field of experience.

Existing Methodology for robustness tests

Robustness stress exists with difference methodology:

- **Step Stress Method**: Application of constraints one by one individually (method described in the chapter below) and step by step:
- **MEOST**: Multiple Environment Over Stress Testing
- **HALT**: Highly Accelerated Life Test [registered trademark]

MEOST:

Le MEOST (Multiple Environment Over Stress Testing) is a test program that combines stresses applied **beyond specifications but within known destructive limits** (defined or previously determined by HALT tests). The combination of constraints and use cases reveals weaknesses in the interactions between the impacts of constraints on the product.

The following parameters can be combined:

- Electrical input signals (Power…)
- Variations in output signals (loads…)
- Operating mode (on/off…)

MEOST is well adapted for intermittent failure detection

HALT: Highly Accelerated Life Test

The HALT method consists in evaluating the robustness versus the temperature constraints, thermal cycles, and shocks & vibrations higher than the use then to cumulate in a second time all the stresses during the same sequence of test. The method is based on a specific HALT oven to apply extreme stresses (limit at -100°C to +200°C), rapid temperature variation (60°C/min…), and vibration up to 60Grms with 6 degrees of freedom. These constraints are applied individually or cumulatively.

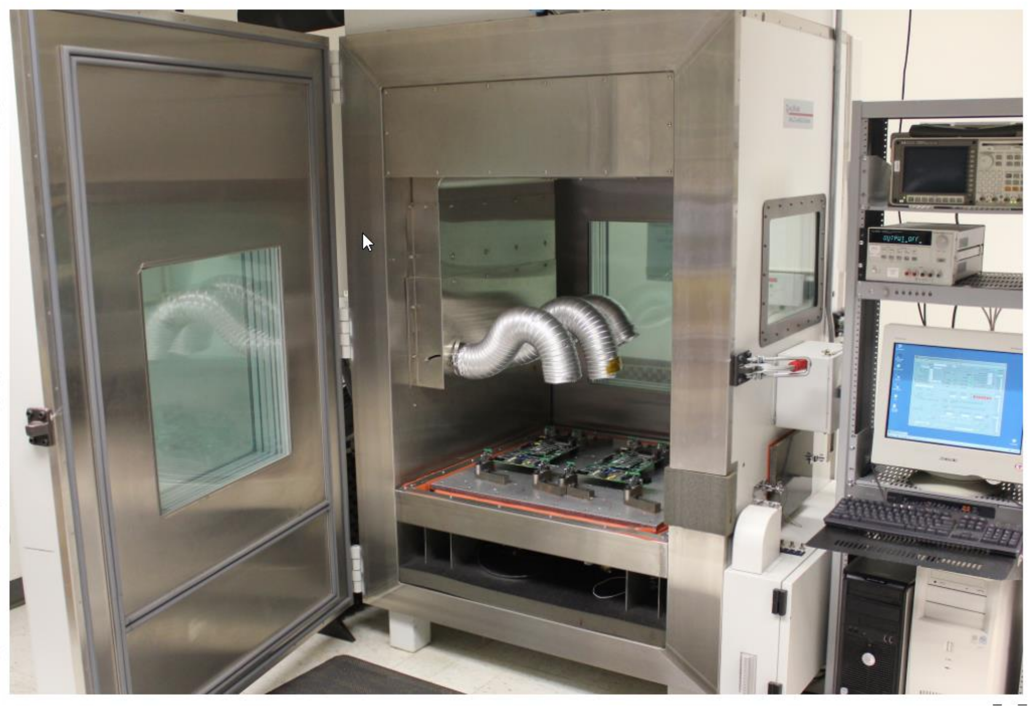


Figure 55: HALT enclosure (source Emitech).

HALT methodology is widely used in aeronautic and military domain

How to conduct a Step Stress Method robustness test

Robustness tests are generally carried out with a single DUT by constraint. These tests can be carried out on conventional test equipment for the constraints applied individually (oven, vibration bench, etc.) or an HALT enclosure.

The proposed method illustrates the philosophy of robustness testing. Everyone can define his method adapted to his product and mixing different methodologies.

Step 1: Select constraints to apply based on the mission profile:

- Low temperature
- High temperature
- Vibration
- Mechanical shocks
- Thermal cycles
- Electrical constraint (Vbat …)
- Etc. …

The relevant constraints to be applied in the test are defined via risk analysis and the main failure modes observed on products in service.

Step2: Setting up a test set up to pilot and monitor the DUT. The objective is twofold:

- To be able to place the DUT in operation as close as possible to the conditions of its life cycle.
- To carry out the most exhaustive monitoring possible (one of the keys to the success of the method) to detect failures as early as possible, even intermittent ones, and to be able to precisely locate the failure (a thermal analysis of the system will identify hot spots in order to place the thermocouples in an optimal manner).

Step 3: Perform the test for each stress individually using the step stressing method.

On the first constraint, the DUT is placed in operational mode by placing all the parameters at the maximum specified values. Then we will increase the selected stress by step until its failure. We usually start with low temperature stress (the least destructive)

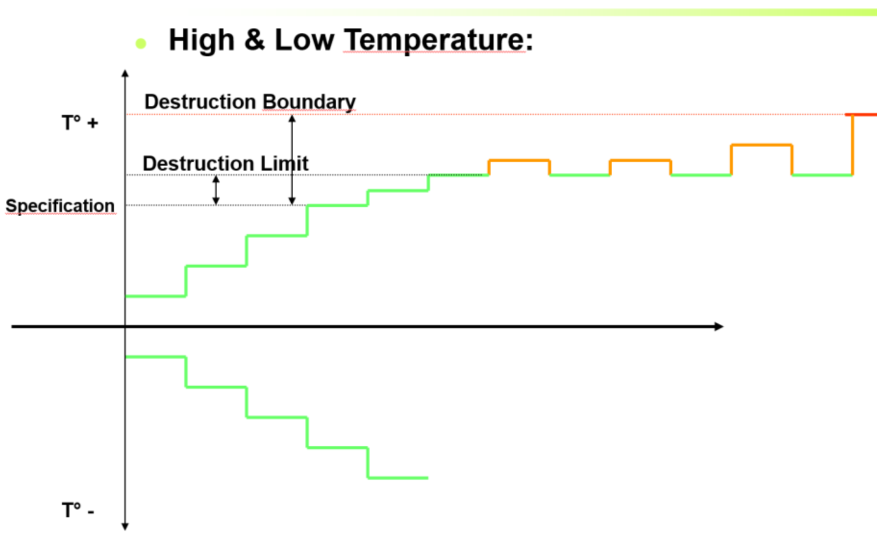


Figure 56: Step stressing method for temperature.

It should be noted that these levels must have a sufficient duration for the stress to apply to a stabilized level. A functional test is then performed to decide the next step. If the test is successful then the stress is changed to the next step. If a failure is reached, then the stress is reduced to the maximum specified value to see if the failure is reversible. We then have two possibilities.

- Failure is irreversible. The destruction limit is reached. A failure analysis is then performed to find the origin of this failure.
- The failure is reversible. The functional limit is reached. We return to the final level reached. We then continue the gradual growth by returning after each step to the maximum allowed value to validate the reversibility of the failure. Once the irreversible failure level was reached, the destruction limit was identified. A failure analysis is then performed to find the origin of this failure.

If it is considered that the level reached is sufficient without generating a failure of the DUT (for example 150% of the maximum specified value), the test can be stopped.

<u>Step 4:</u> Failure analysis identifies the weak point of the system versus this constraint. 2 possibilities arise:

- The operating and destruction limit is acceptable for the mission profile. They make it possible not to precipitate failures in case of punctual drifts beyond the limits specified on the selected stress
- Operating and destruction limits are not acceptable. Corrective action is identified. It allows to build the robustness of a product and possibly exceed the constraints of its life profile to further increase its reliability.

The failure can also be corrected (component replacement...) in order to continue the test to identify the 2nd functional or destruction limit related to another factor (2nd weak point). It is important to have a system expert on site during the test so that failure analyses can be performed quickly.

<u>Step 5:</u> Perform the same mode of operation on another constraint, and so on in order to identify the functional limits & destructions for each constraint.

How to interpret the results of robustness tests?

Robustness tests allowed to detect weak points and evaluate functional or destructive margins.

- Each functional or destruction margin will need to be risk-assessed to determine whether it is acceptable or not. The following parameters should be taken into account:
- Dispersion related to the manufacturing process
- Dispersion related to product variability (dimensional...)
- Dispersion related to component variability
- Variability of mission profile (user, environmental...) 1 system in the engine compartment shall not have the same margin level as in the passenger compartment)
- Number of field applications?
- Etc.

The comparison of functional or destruction limits on different generations of previous product or field feedback data is also a key aspect in this risk analysis to identify the results at the level of functional & destruction margins with the Nature & the return number market...

The difficulty of obtaining a precise and comprehensive mission profile makes it necessary to have significant margins in order to improve product robustness and reduce the incidence rate in field return

The suggested margins (derating) can be read in literature [3]:

- Derating for mechanics: 50%
- Derating for electronics: 40%

Robustness test in series production & serial life phase

There are 2 types of process based on robustness tests during series production:

- HASS (Highly Accelerated screening test)
- Periodic sampling robustness test

HASS in serial production

Highly Accelerated Stress Screening (HASS) is a 100% screening technique that stresses systems to levels of severity derived from HALT tests. The time required to carry out these tests makes it unsuitable for high volume production at 100%. One possible adaptation to the constraints of high volume production may be to carry out sampling to condition the delivery of each batch of production.

The objective is to precipitate defects related to the manufacturing process before delivery to the customer.

The HASS profile is in two stages:

- Precipitation step during which stress levels are applied above the functional limits but below the destruction limits (sufficient margin between the operating limit and the destruction limit is essential). The objective is to transform latent defects into obvious defects (detectable defects)
- Detection step with sufficient exposure times to high and low temperatures to perform the proper functioning tests and discover failures.

The HASS parameters will have to be validated through a Proof of Screen in order to validate that the HASS does not impact the overall reliability of the system.

Robustness test by sampling

Periodic robustness test by sampling: The objective is to detect deviations on products related to the process or product in production phase.

Several processes can be set up depending on the desired objectives. One can set up the following process:

- Perform robustness tests at a defined frequency (1 time/ month, 1 time per quarter, 1 time/ half-year...) on 1 product taken in production. The result of this test (functional limit & destruction limit) is compared with the results of previous tests to detect possible deviations. In case of doubt, a second test can be carried out to eliminate the risk of an atypical product.

This monitoring can be done by technology family in order to limit the number of references to be evaluated.

Practical sheet references

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[3] *World Class Reliability: Using Multiple Environment Overstress Tests to Make it Happen* - R Bhote & K Bhote. - American Management Association - 2004.

Bayes Theory and the Markov Chain Monte Carlo methods

Bayes Theorem

Bayes theorem is at the root of the Bayesian MCMC framework. In its useful form:

$$p(\theta|Data) = \frac{p(Data|\theta) \times p(\theta)}{p(Data)} = \frac{p(Data|\theta) \times p(\theta)}{\int_{\theta_{min}}^{\theta_{max}} [p(Data|\theta) \times p(\theta)] d\theta} \qquad eq(1)$$

$$p(\theta|Data) \propto p(Data|\theta) \times p(\theta) \qquad eq(2)$$

With:

- $\theta = (\theta_1, \theta_2, \ldots \theta_p)$: a set of parameters to be studied, for example Weibull shape and scale parameters (β, η), or Log Normal location and scale parameters (μ, σ)
- $p(Data|\theta)$: this is the Likelihood of the data given the set of parameters θ. Frequentist Statistics solely analyses these quantities.
- $p(\theta)$: this is the prior probability distribution of parameter θ
- $p(Data)$: a constant also called the marginal likelihood, because it is marginalized through all possible values of parameter θ, and weighted with the prior.
- $p(\theta|Data)$: this is the posterior probability distribution of the set of parameters θ given the data that was analyzed. The confidence of the parameter θ is directly linked to the posterior distribution probability density

In the second form, the Bayes theorem describes the proportional relationship between the posterior distribution of the set of parameters θ on the left side, and the product of the prior on θ multiplied by the likelihood of the data given θ.

With the hypothesis of independence between parameters, then eq (2) becomes:

$$p(\theta|Data) \propto p(Data|\theta) \times p(\theta_1) \times p(\theta_2) \times \ldots \times p(\theta_p) \qquad eq\ (3)$$

This third form is useful for computation with Formal Analysis, or Grid Approximation.

In Formal Analysis, the rule is to use of a likelihood and a priori with the same general form. This kind of a priori is called a Conjugate Prior. This field of Bayesian studies has been the most developed in the past decades since computer did not have the power to assess complex conditional probabilities when dealing with multiple parameters.

In Grid approximation, computation of a "point to point" value of $p(\theta|Data)$ following eq (3) is performed. However, it remains limited to simple problem solving. That is the reason why Markov Chain Monte Carlo algorithms are more widely used nowadays.

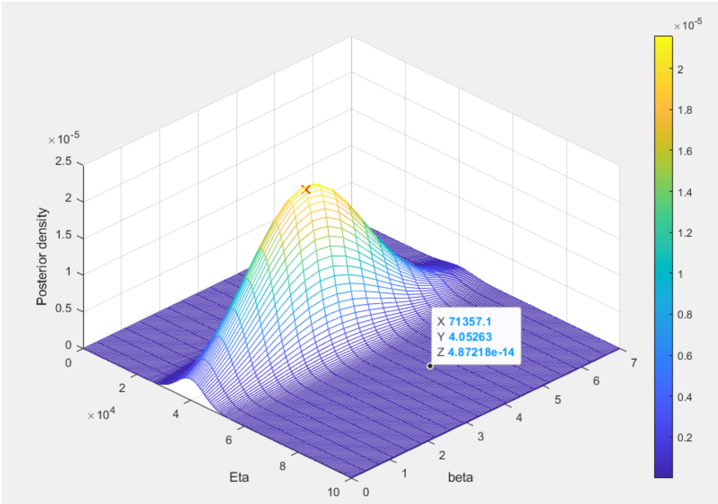

Figure 57: An example of Grid Approximation on time to failure data according to a Weibull likelihood, with Gauss priors on Shape and Scale parameters.

Markov Chain

A "Markov Chain" is a method for generating a sequence of random variables where the current value probability only depends on the value of the immediate prior variable. Any such process, in which each step has no memory for the states before the current one, is called a first order Markov process. The succession these steps is called a Markov Chain.

Given:

$$X = (X_1, X_2, \dots, X_k, X_{k+1}, \dots X_n) \quad eq(4)$$

X is called a first order Markov Chain if for all k:

$$P(X_{k+1}|X_1, X_2, \dots, X_k) = P(X_{k+1}|X_k) \quad eq(5)$$

Understand here that the n elements of the Markov Chain are an iteration. This means that the higher is k among all n elements, the more the simulation is reaching through its end.

Monte Carlo Simulation

Any simulation that samples many random values repeatedly from a distribution is a Monte Carlo Simulation.

In most problem solving with Monte Carlo, there is a mathematical relationship between variables$(X_1, X_2, \dots X_p)$ and a response Y. Monte Carlo needs a deterministic model.

Each variable has a known distribution with specific parameters. The goal is then to draw the response Y, and if possible, draw its distribution.

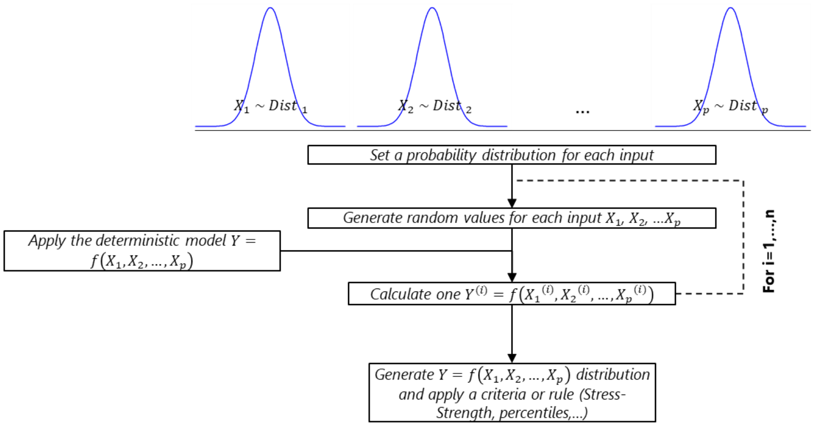

Figure 58: General form of a Monte Carlo Simulation.

Some criteria on Y can then be applied. A typical example of Monte Carlo simulation for Reliability is the Stress Strength methodology, where X_1 is the Stress with its own distribution (e.g $Log\ Normal\ (\mu_1,\ \sigma_1)$, and X_2 the Strength $Weibull\ (\eta_2,\ \beta_2)$.

Then the simulation is drawn several times for the simultaneous values of both X_1 and X_2. We then count the number of occurrences when $X_1 > X_2$, or when the $Stress > Strength$. In this case, it would be the criteria to check (ppm level).

Markov Chain Monte Carlo (MCMC)

Calculating a quantity from a probabilistic model is referred to a probabilistic inference.

The direct calculation of the desired quantity from a model of interest can be challenging. That's why the expected probability must be approximated.

Bayesian calculations require integrating over possibly high-dimensional probability distributions to make inference about model parameters or to make predictions. It needs to integrate over the posterior distribution of model parameters given the data.

One solution is to draw independent samples from the probability distribution, then repeat this process many times to approximate the desired quantity. This is the principle of Monte Carlo sampling.

The problem with Monte Carlo sampling is that it does not work well in high-dimensions. Then, Monte Carlo sampling assumes that each random sample drawn from the target distribution is independent which is not the case for Bayesian structures.

The solution to sampling probability distributions in high-dimensions is to use Markov Chain Monte Carlo (MCMC). MCMC enables to draw samples from the posterior distribution (see Bayes theorem) by constructing a Markov Chain. As the sample size gets larger, the Markov Chain converge to the actual posterior distribution of the parameters.

MCMC will run a random process throughout all possible values of each parameter posterior distribution. The most famous one to date is the «Random walk» process.

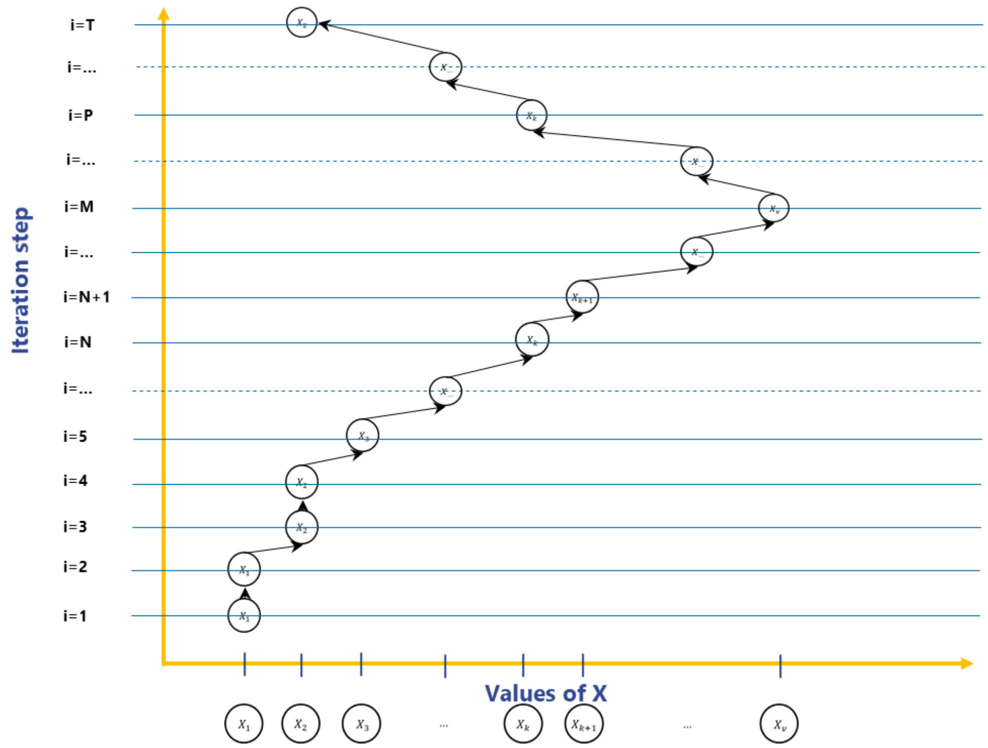

Figure 59: Random Walk process drawing the posterior distribution of a given parameter.

A Random Walk is a probabilistic process describing a path that consists in a succession of random steps on some set of possible values.

The number of chains involved, and the length of each chain, will have an influence in the quality of the results. After running through every chain length, the Random Walk will draw a histogram of distribution. Once this histogram is available, it becomes easy to normalize in order to draw a probability density function (see fig below). When applying Bayes Rule, the Random Walk will approximate the same density as the parameter posterior.

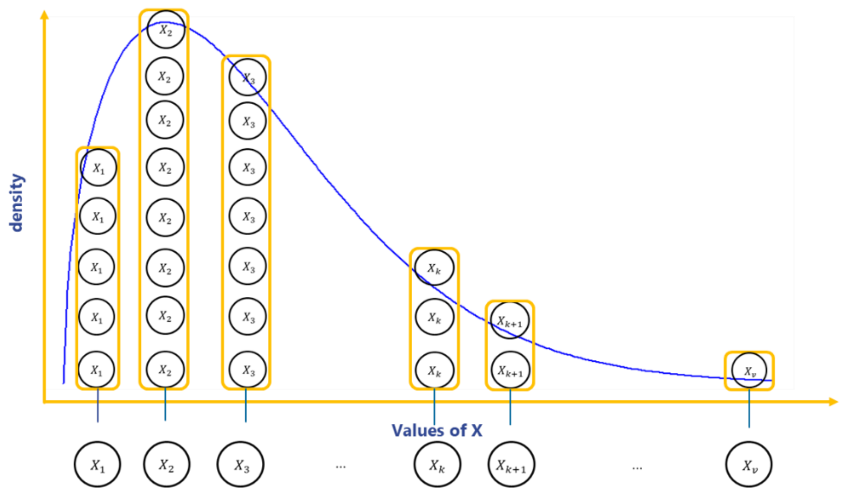

Figure 60: Drawing the posterior density of a given parameter through MCMC.

Gibbs Sampling

Gibbs Sampling is a particular case of MCMC methods, and more precisely, it is a particular case of the broadly used Metropolis Hastings algorithm.

Metropolis-Hastings enables to draw samples from several types of a priori and likelihoods, with little restrictions on it. However, Metropolis-Hastings have the drawback to be dependent on a proposal density. Computation can be hardly achievable depending on the choice of this proposal density.

Gibbs sampling operates following the mode below.

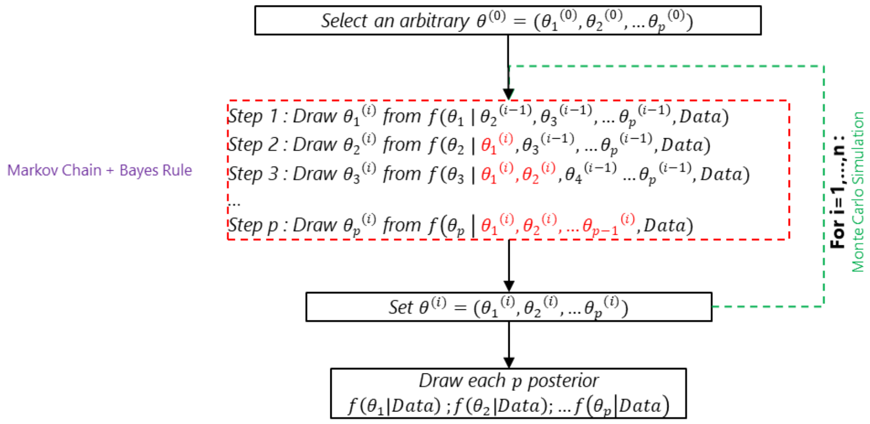

Figure 61: Illustration of the Bayes Rule combined with the MCMC Gibbs sampler.

Gibbs samplers always follow the ascending order of the p parameter vector of interest. It will first draw the first parameter, then the second, until the final p^{th} parameter.

As a reminder:

- $\theta = (\theta_1, \theta_2, \dots \theta_p)$ is a p dimensional vector of parameters of interest
- $f(\theta| \, Data) \propto Prior(\theta) \times Likelihood(Data|\theta)$
- $f(\theta| \, Data)$ and $f(\theta_k| \, Data)$ are density functions, thus their integration through all possible values of θ or respectively θ_k, will equate to 1.

Practical Usages of the Markov Chain Monte Carlo methods

Gibbs sampling with R & JAGS

As seen above, Gibbs Sampling requires to compute the conditional probabilities, for each parameter of interest, through the data and all other parameters. This can be challenging if performed by hand calculation. To get rid of this burden, the use of the JAGS MCMC generator that will provide chains according to specified a priori and likelihood, can be proposed. In the ***Figure 62*** below, the communication with JAGS is performed through R programming language. The choice of this language, with respect to others that would perform equivalent tasks, is justified by the following considerations:

- Free language
- Broad availability of publications and scientific community.
- Easy to quickly develop skills with MCMC computation.

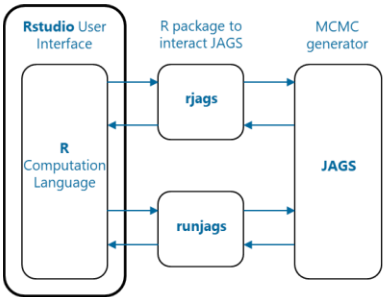

Figure 62: Proposal for an R setup to compute MCMC with a Gibbs sampler.

Rjags is a library containing a set of basic functions allowing you to communicate with JAGS, to generate MCMC chains. **Runjags** contains more advanced functions, notably allowing Bayesian calculations to be parallelized. If desired, the reader is invited to refer to the help for each of these libraries, in order to understand their content.

JAGS is the MCMC chain generator. It therefore contains the Bayesian model, including the likelihood of the data and the prior choices. Depending on the model chosen, the JAGS file can become complex, as in the case of hierarchical structures with "hyper" a priori. It is this file which contains the "Bayesian knowledge/Prior knowledge" part of the user. It is therefore the centerpiece of the simulation.

Creations of MCMC chains, Diagnostics and Output Analysis

With the usage of modern computing languages, creating MCMC to solve Bayes problem assessment became fluent since the end of the years 2000. However, there are still diagnostics that MUST be performed to check if the simulation performed well. Moreover, one must not be dependent on a methodology for the sake of simplification. Instead, each created algorithm, choice of priors, likelihood, or the Bayesian structure, must be carefully thought by the person running the MCMC computation. For this part, no formal guideline exists.

But for the diagnostics, there are some rules and tips that should help. MCMC computation should target 3 main goals:

- **Representativeness** of the posterior distribution. As the MCMC chains are performing a Random Walk throughout the set of all possible values, one must make sure that each chain went through all of these values and were not influenced by arbitrary initial values.

- The chains must be **Accurate and Stable.** There must be enough chains with sufficient size to achieve this target. Each parameter, quantiles, central tendency and Credible Limits must be influenced as little as possible by the choice of seeds, states, pseudo random numbers or simply restart of simulation
- **Efficiency**. Once the computation of Bayesian analysis becomes fluent, the programmer must focus on making the MCMC generation efficient.

Posterior representativeness

Because of the initial conditions and the Random Walk, the chains can diverge from the posterior distribution they were intended to draw. To solve this, MCMC computations use **Burn-in** periods that simply remove the first iterations of each chains. These iterations are usually a few thousands of initial steps. But they can be lowered to hundreds of steps in the best conditions.

Visual Inspection and checks:

- M**ultiple chains and plotting**: plotting several chains helps since any deviation of one chain compared to others will be an indication of a bad simulation.
- **Trace plot of posterior**: Plotting each posterior will help since an awkward shape in the posterior or an unusual Shrinkage of the posterior is an indication that something could be wrong.

Some numerical checks may help finding the good set of parameters for the simulation:

- **Shrink Factor (Gelman-Rubbin)**: This parameter measures how much variance there is between chains, relative to the variance there is within chains. If all chains are representative and converged to their expected values, the average variance between the chains should be equal to the average variance within chains. The Shrink Factor must converge to 1. Any result above 1.1 could indicate that one chain is stuck or the length of the chains is insufficient.
- **Autocorrelation (ACF)**: This is the correlation in each chain, relative to a k step ahead a translation of the chain. These k step translations are called "lag". The Autocorrelation function of MCMC is simply the correlation measurement through the specter of lags. For each step k ahead, there is a different correlation between the chain steps. Said in a simpler way, the correlation of the data with the k first steps removed (since the chain was translated ahead of its current position), is measured relative to the original set of data. This number must converge as soon as possible, and the convergence value must be as small as possible (0 preferably).

Accuracy and Stability

For Accuracy and Stability, checking of the previous parameters with the same visual inspection as for the Representativeness topic is advised. There are however some mathematical tools that enable to check for it.

- **Autocorrelation (ACF).** As for the previous goal, one must check this function to ensure accuracy and stability of all chains. ACF can be reduced through thinning technique, that consists in storing only 1 on k^{th} step data of the chain. For example, taking a thinning of 100, leads to storing the 1st, 101st, 201st… step of the chain. But high thinning will make chains less stable and accurate. One must therefore balance the weight of thinning in the simulation.

- **Effective Sample Size (ESS)**. This measurement consists in dividing the chain total length by the amount of autocorrelation.

$$ESS = \frac{n}{(1+2.\sum_{k=1}^{\infty} ACF(k))}$$

  Usually, consider that Accuracy and Stability are fulfilled when $ESS >= 10\,000$
- **Monte Carlo Standard Error (MCSE).** To ensure accuracy, this quantity must be as small as possible.

$$MCSE = \frac{Samples\ Standard\ Deviation}{\sqrt{ESS}}$$

Efficiency

This final point of focus should be performed once MCMC techniques are understood.

- The easiest way is to perform parallel computing of the chains, if the programming language enables it. In R, this can be done through the package **Runjags**.
- Use appropriate priors and likelihoods. Between two sets of priors-likelihoods that enable equivalent results, go for the one that makes calculation faster.
- Try a reconfiguration of the Bayes JAGS code. For linear regressions, it can be useful to work for example, with standardized data relative to their mean and standard deviation.
- Choose the sampler and the MCMC technic that provides equivalent results in a less computer intensive time Gibbs Sampling with JAGS sampler are presented here for the purpose of simplicity and general purpose fit. Other samplers and languages may best fit your needs, such as technics with Hamiltonian Monte Carlo with STAN sampler, or other computing languages.

Visual Summary

Comparing a weak vs a good simulation could lead AT LEAST to one of the four difference depicted below:

Figure 63: Visual inspection of the 4 main indicators, marking the difference between a good and a bad simulation.

It is important to underline that a single bad indicator could lead to an inspection or correction of the complete simulation. Errors of convergence and stability may be due to initialization problems, or "thinning", while the Bayesian kernel of the sampler is correct.

Risk Assessment

With Bayesian statistics, there is no "Confidence Intervals" as for the rules defined for the Frequentist statistics (e.g. Student distribution to assess the risk on the mean of Gaussian distributed data). Instead, there are "Credible Intervals" that are directly linked to the posterior distribution.

Two kinds of intervals are used for risk assessment:

- **Equally Tailed Intervals (ETI)**. By default, these are the intervals computed through the R command "summary". From the posterior distribution, ETI provide a same risk quantity on the lower side and on the upper side of the distribution. They can therefore be thought as "symmetrical risk" as the lower and upper bound risk with Frequentist statistics. ETI are easy to implement (since command lines are available). ETI are also insensitive to reconfiguration, which is an interesting property when you need to change the configuration of the problem (log, exp, Z standardization…).

- **Highest Density Intervals (HDI).** These intervals draw the risk directly from the posterior distribution, by selecting the area of the density that contains the most credible values, given the required risk value. It can be retrieved by sorting the sampled data and using an iterative process to find the shortest interval of credible values.

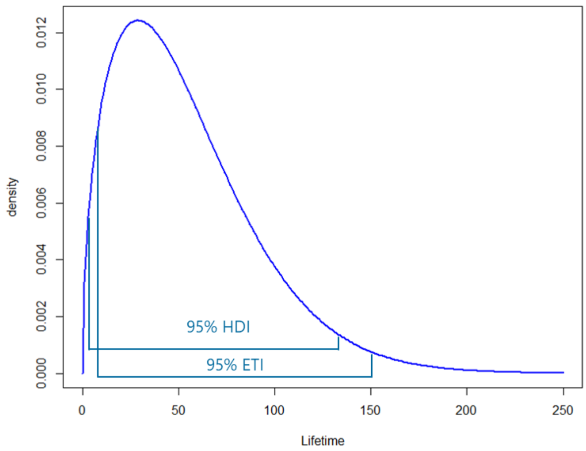

Figure 64: Difference between HDI and ETI.

Application: Time to Failure Analysis with censored data – Reliability Determination testing

The application below deals with test bench failure samples including suspended data. This type of scenario is often encountered in Reliability verification or validation testing.

The raw data for the example is below. "0" means the data is not censored and "1" means right suspension

Table 33: Data for the MCMC example with suspensions/censored data.

Time to Failure	Censored
21	0
33	1
40	0
66	0
70	1
84	0
100	1
110	1
150	0
200	0

This example uses a Weibull likelihood for times to failure, and several a prioris that are more or less vague.

The following can be observed: when the prior is too diffuse, then the posterior law is more consistent with the likelihood distribution

```
# Diffuse prior case

    shape ~ dgamma(1, 1) # mean = a/b ; variance a/b^2
    scale ~ dgamma(1, 0.01)
```

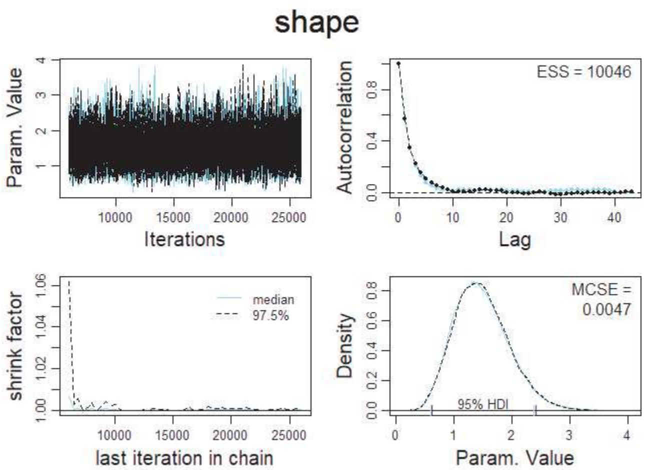

Figure 65: Posterior distribution on the shape parameter with diffuse priors.

Conversely, when the prior is narrower, then the posterior distribution is strongly impacted by the prior probabilities. The likelihood distribution can sometimes become ineffective on the posterior distribution.

```
# Narrow prior case

        shape ~ dunif(1.5, 5)
        scale ~ dunif(80, 400)
```

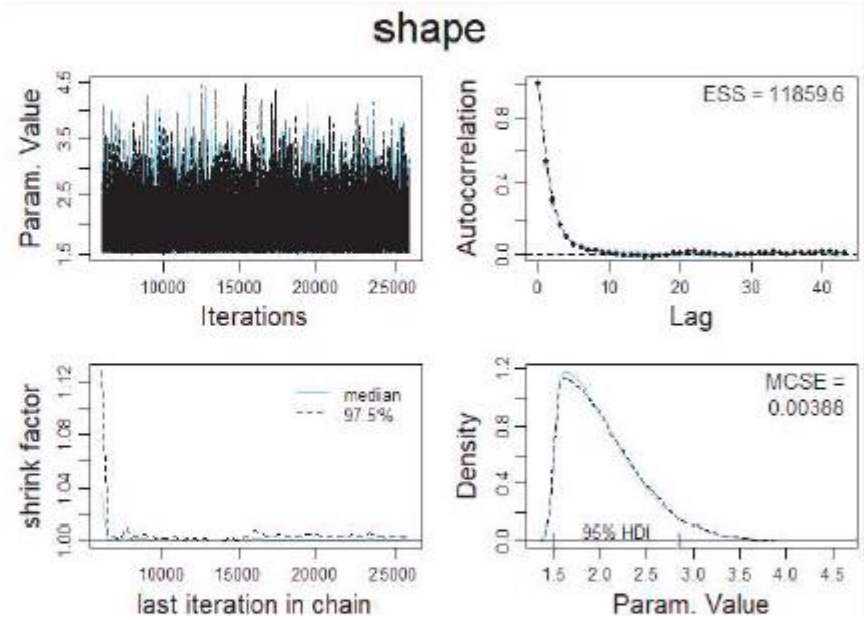

Figure 66: Posterior distribution with a narrow prior.

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Practical sheet 14: Determination of the thermal cycling endurance test of a power computer

Background

A Power Electronic Unit (PEU) is an energy converter (Charger, DCDC or Inverter) with a power of more than 1kW cooled by a heat transfer fluid circuit.

These PEU are generally composed of a power stage ("Power Module"), a control stage ("Driver Card") for the power elements and a control stage ("Control Board") ensuring complete management and communication with other ECUs. These PEU can be strongly integrated with multiple conversion functions at the same time (e.g. OBC-DCDC) to minimize the number of cards and volume at the expense of modularity.

The substrates of electronic boards can be composed of different materials (e.g. FR4 for a "Control Board" and SMI for the "Power Module") supporting the electronic components. The PEU can integrate several power filtering elements (capacitors, inductors or transformers) as well as high power (bus-bars) and low power (CAN) connections.

All of this forms a heterogeneous assembly in terms of mass and thermal inertia with electronic boards (PCBAs) of a few tens to hundreds of grams, inductors or transformers of the order of one kg and a complete assembly with its aluminum housing of the order of ten kg.

This assembly, subject to thermal variations, can have different failure modes resulting from:

- Stresses on solder joints and metal interfaces of electronic components,
- Stresses on assembled printed circuit boards (PCBAs) due to a coating or molding agent,
- Constraints on wire-bonding (in electronic components and power modules),
- Losses of tightening torque in mechanical or electrical assemblies,
- Degradations of contacts in inter-card connections,
- Damages to the waterproofing (cooling circuit or housing closing cover), etc.

Glossary

AMR	Absolut Maximum Rating
DCDC	Direct Current to Direct Current converter
ECS	Equivalent Cumulated Stress
IGBT	Insulated Gate Bipolar Transistor
OBC	On board Charger
PCB/PCBA	Printed Circuit Board / PCB Assembled
FEW	Power Electronic Unit
PFC	Power Factor Corrector
GMV	Fan Motorcycle Group
TC	Thermocouple

Methodology

Given the nature of the different failure modes, **thermal cycling endurance tests** can be done at multiple levels and in several steps, with the same methodology.

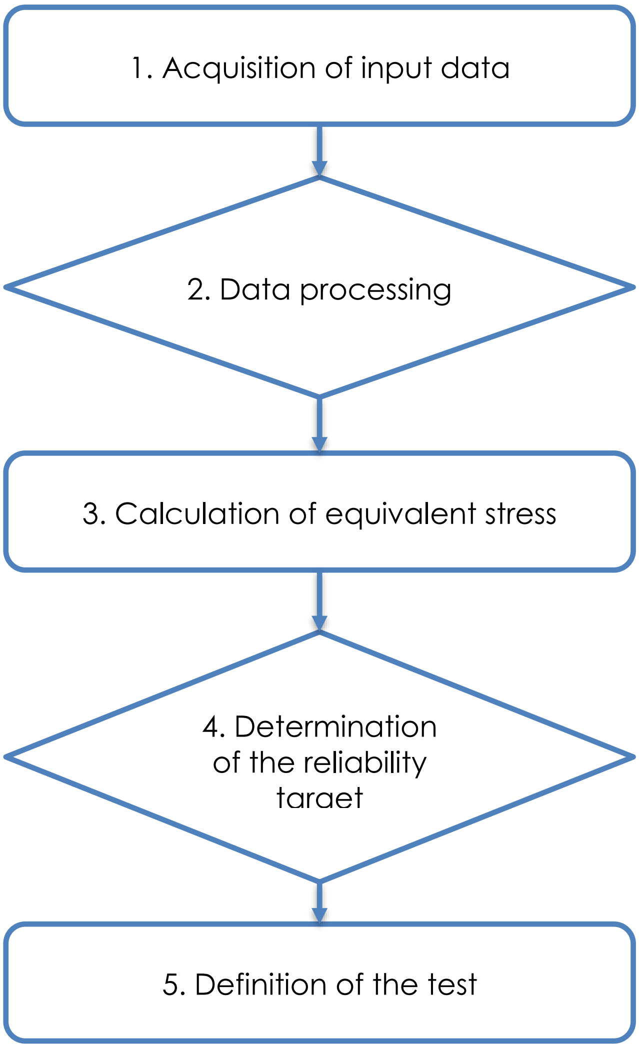


Figure 67: Macroscopic flowchart of the test sizing.

Power Modules, and passive power components with high thermal inertia due to their volume, will be treated separately from the assembly of the electronic boards.

Power Modules are validated by suppliers within the framework of standards such as the AQG324.

Passive power components are validated by suppliers based on the AEC-Q200 standard for the qualification of "traditional" passive components.

For the validation of a PEU in the context of an endurance test in thermal cycling, the damage criterion is defined by a number of cycles at a given temperature variation.

<u>E.g.</u> 1000 cycles from -40°C to 100°C (ΔT=140°K)

Most of the time, the temperature variation taken as a reference is the maximum possible for the test.

In the case of power electronics and electric motor, it is necessary to separate the temperature variation due to self-heating from the external temperature variations (daytime cycling or other components in operation that modify the external environment (air or water)).

Thus, the definition of the **number of thermal cycles** under test with air or coolant is independent of the definition of the **number of activations** of the PEU during the test.

Ex:

- 40000 activations of 1min ON/1min OFF (ΔT=30°K at startup) during

- 800 thermal cycles from -30°C to +60°C air and coolant (ΔT=90°K on mechanical parts)

The acceleration law used for this endurance test is the Coffin-Manson law:

$$\text{Acceleration Factor} = \frac{N1}{Ne} = \left(\frac{\Delta Te}{\Delta T1}\right)^n$$

- N1 is the number of reference cycles from the mission profile
- ΔT1 is the amplitude of the referenced thermal cycles
- Ne is the number of accelerated cycles for the test
- ΔTe is the amplitude of the accelerated thermal cycle, generally Tmax-Tmin in storage for the passive phases or Tmax_on – Tmin_off for the active phases that will take into account self-heating.
- n is the coefficient of the acceleration factor. It depends on the failure mode and the materials under test. Historically, it is between 2.5 and 3 to validate electronics as a mixture of different materials to be validated: soldering, varnish, plastics, bonding of electronic components, PCBs.

Table 34: Values of the Coffin-Manson coefficients as example.

Material	m
Leaded-Solder – General Use	**2.5**
Lead-Free Solder (97Sn/3 Ag & 91 Sn/9 Zn)	**2.4**
Cu and Lead frame alloy (TAB)	2.7
Al wire bond	3.5
Au4Al fracture in wire bonds	4
PQFP Delamination /Bond failure	4.2
Copper	5
Au wire Downbond heel crack	5.1
ASTM 6061 Aluminum alloy	6.7
Alumina fracture-bubble memory	5.5
Inter layer Dielectric cracking	5.5 ± 0.7
Silicon fracture	5.5
Si fracture (cratering)	7.1
Thin Film cracking	8.4

Procedure

The input data can be summarized by obtaining 3 mission profiles and their subsequent processing, to arrive at 3 equivalent cumulative stresses (ECS) that can be added up during the thermal cycle endurance test.

There are therefore three types of mission profile:

- **the activation mission profile.**
- **the mission profile of the cooling system.**
- **the environmental mission profile in passive mode.**

Activation mode stress phases

Acquisition of input data

Construction of the **activation mission profile** corresponding to the power activation phases of the PEU.

It will be built mainly to characterize the thermal stresses of the power stages (PFC, DCDC, Power Module) but also to characterize the internal self-heating of a complete PEU. For obvious reasons of cost, it is not possible to take temperature measurements on a large customer panel that is representative of the entire population. On the other hand, it is possible to build a thermal model of the power stage and to build a mission profile of the stage or component from numerical simulations or to make a thermal characterization on a prototype PEU.

Ex:

- *A PFC diode in a charger can have its temperature vary and cycle by a few degrees with a period of 20ms (50Hz).*
- *The IGBTs of a "Power Module" in an inverter can vary by several tens of degrees during an acceleration under load (increase in current).*

Table 35: Customer rolling profile 63% for a considered electric GMP inverter[10].

Number of driving phases sorted by output battery current and time duration										
6 % of Coverage		Time Class (s)								
		[0...1]	[1...5]	[5...20]	[20...60]	[60...300]	[300...1200]	[1200...3600]	[3600...7200]	[7200...1e+30]
Current Class (A)	[0...20]	0	0	11337	22085	28519	20658	4053	641	256
	[20...40]	0	0	0	0	2625	9066	9944	1138	445
	[40...60]	0	0	0	0	194	797	1795	264	0
	[60...80]	0	0	0	0	0	65	463	0	0
	[80...100]	0	0	0	0	0	45	0	0	0
	[100...120]	0	0	0	0	0	0	0	0	0
	[120...140]	0	0	0	0	0	0	0	0	0
	[140...160]	0	0	0	0	0	0	0	0	0
	[160...180]	0	0	0	0	0	0	0	0	0
	[180...200]	0	0	0	0	0	0	0	0	0

Number of activations = f(time class; current class)

[10] The tables and graphs presented are only partial and/or incomplete illustrations.

Data processing

In the case of an inverter, current-time rolling profiles make it possible to determine temperature variations on the thermally modelled stage.

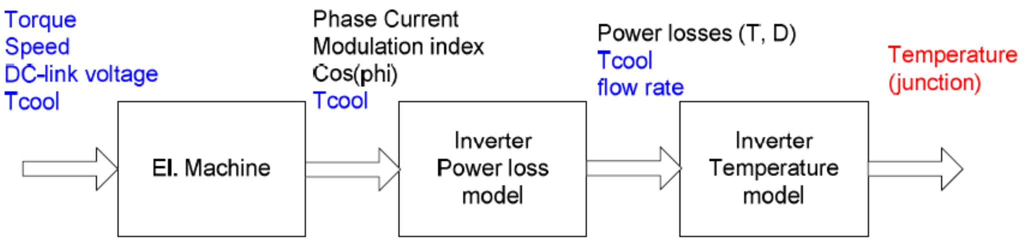


Figure 68: Flowchart of the junction temperature calculation.

After passing through the supplier's thermal model either of the "Power module" or of a power component, or via a thermal characterization of the component (e.g. transformer), or of the PEU, we obtain the 2D table of self-heating by activation time-current.

Table 36: Table 2D of self-heating by duration-current activation.

Time Class

Current class	[0…1]	[1…5]	[5…20]	[20…60]	[60…300]	[300…1200]	[1200…3600]	[3600…7200]	[7200…1e+30]
[0…20]	0,0	0,0	0,0	0,0	0,1	0,7	1,9	2,4	2,5
[20…40]	0,0	0,0	0,0	0,0	0,3	2,1	5,7	7,1	7,4
[40…60]	0,0	0,0	0,0	0,1	0,6	3,5	9,5	11,9	12,3
[60…80]	0,0	0,0	0,0	0,1	0,8	4,9	13,3	16,6	17,3
[80…100]	0,0	0,0	0,1	0,1	1,0	6,3	17,1	21,4	22,2
[100…120]	0,0	0,0	0,1	0,2	1,3	7,7	21,0	26,1	27,1
[120…140]	0,0	0,0	0,1	0,2	1,5	9,1	24,8	30,9	32,1
[140…160]	0,0	0,0	0,1	0,2	1,7	10,5	28,6	35,6	37,0
[160…180]	0,0	0,0	0,1	0,3	1,9	11,9	32,4	40,3	42,0
[180…200]	0,0	0,0	0,1	0,3	2,2	13,3	36,2	45,1	46,9

Table: Self-heating = f(time class; current class)

Calculation of equivalent stress

From the 2 previous 2D tables, for each pair (number of activations/self-heating), representative of the reference cycles, the equivalent number of cycles for a fixed test ΔTe is calculated.

Example of defining test conditions:

Tmin Coolant = -30°C, Tmax Coolant = 65°C giving ΔTe = 95°C

We can calculate the equivalent number of cycles for each couple and sum them up at the end.

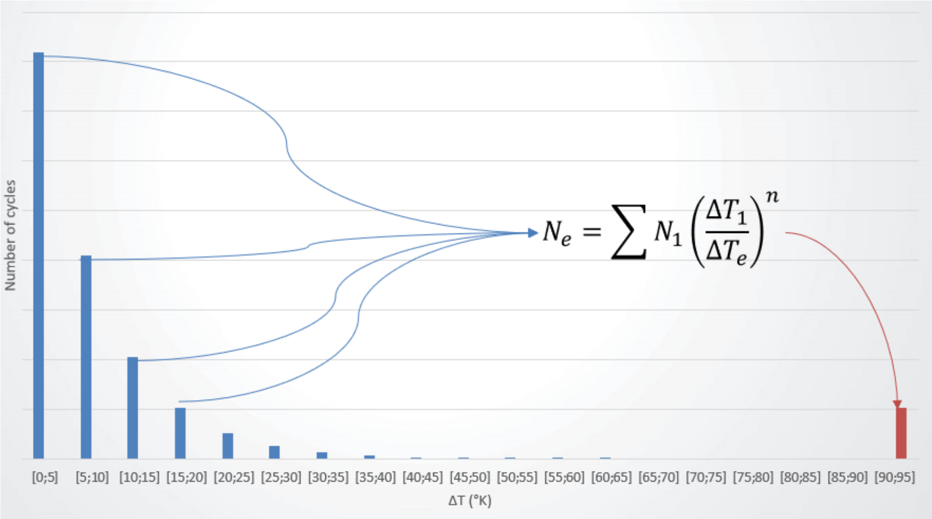


Figure 69: Methodology for calculating the number of equivalent cycles.

In this example, with n=3, from the **mission profile in activation,** the result of this processing gives an **equivalent cumulative stress** (Ne) of 7 cycles, for a ΔTe of 95°C.

Phases of thermal stress induced by the cooling circuit.

During the charging, driving or wake-up phases (ON accessory parking mode) depending on the fan control strategy, the different PEU can be used differently and thus heat the circuit and therefore induce thermal stresses on the other PEU which are either inactive or in standby mode (The PEU being generally connected to the same cooling circuit).

Ex: A charger will heat the cooling circuit which will only be regulated at high temperature to minimize the noise generated by the fan and thus induce a temperature increase in the DCDC which will only work at low load but also the inverter which will be in standby mode.

Acquisition of input data

Construction of the mission profile of the cooling circuit corresponding to the activation phases of the PEU.

During a thermal characterization of the PEU in pre-development, an influence factor is calculated to determine the cooling efficiency of the ambient air and the liquid cooling circuit. The results of this characterization show that coolant generally has more impact than air temperature. This is the objective when designing a part with liquid cooling.

Therefore, to simplify the calculations, only the mission profile of the coolant is considered in this case.

Measurement campaigns on a similar vehicle or on a prototype are carried out to acquire cooling profiles on the road of a customer considered as a reference.

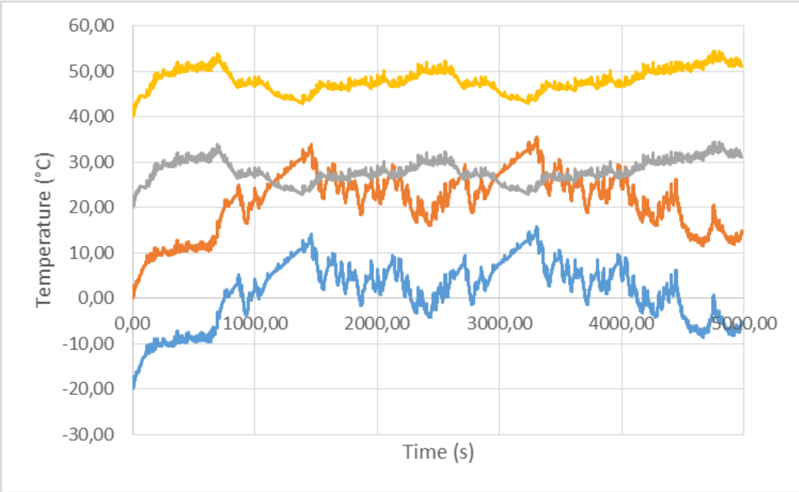


Figure 70: Cooling profiles during driving.

In this example, the cooling profile lasts 4980 seconds (1hr 23mn). The four profiles have different starting ambient temperatures: -20°C, 0°C, +20°C, +40°C.

Given the geographical location of markets – hot country – a **histogram of the water temperatures in "hot country" in driving mode** is considered in the final treatment.

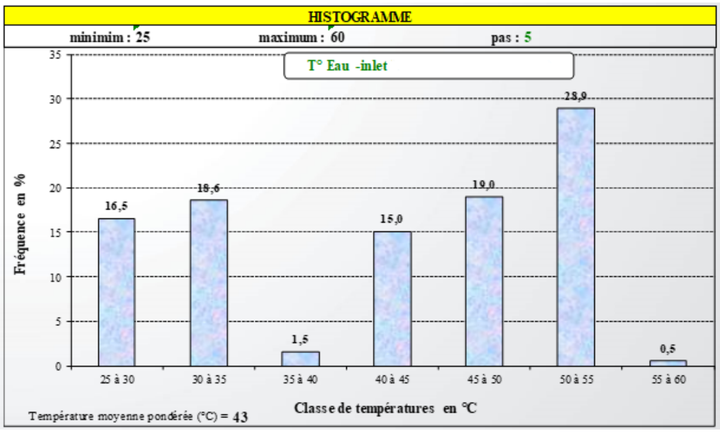
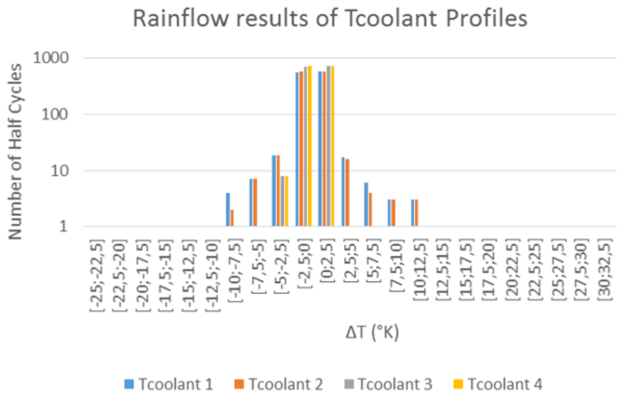


Figure 71: Histogram of coolant temperatures during driving "hot countries".

Data processing

The 4 thermal profiles are broken down by the "Rainflow" method into the number of cycles sorted by temperature range class and by steps of 2.5°C.

Table 37: Mission profile recorded for 4980 seconds.



Class (°K)	Nb Cycles			
	Tcoolant 1	Tcoolant 2	Tcoolant 3	Tcoolant 4
[0;2,5]	571	574,5	719,5	723,5
[2,5;5]	18	17,5	4,5	4,5
[5;7,5]	6,5	5,5	0	0
[7,5;10]	3,5	2,5	0,5	0,5
[10;12,5]	2	2	0,5	0,5
[12,5;15]	0	0	0,5	0,5
[15;17,5]	0	0	0	0
[17,5;20]	0,5	0,5	0	0
[20;22,5]	0	0	0	0
[22,5;25]	0,5	0,5	0	0
[25;27,5]	0	0	0	0
[27,5;30]	0	0	0	0
[30;32,5]	0	0	0	0

Number of cycles (records) = f(coolant temperature, temperature range class)

To project this distribution to the life of the vehicle, it is necessary to consider the customer 63% of the population, driving 7650 hours during the life of the vehicle (382 hours for 20 years).

In this case, each number of cycles must be multiplied by 5530.

N1 = "Nb cycles" * (7650 / (4980/3600))

Table 38: Number of cycles of customer 63%.

ΔT1 (°K)	N1			
	Tcoolant 1	Tcoolant 2	Tcoolant 3	Tcoolant 4
2,5	3158333,0	3177692,3	3979720,8	4001845,8
5	99562,2	96796,5	24890,5	24890,5
7,5	35953,0	30421,8	0,0	0,0
10	19359,3	13828,1	2765,6	2765,6
12,5	11062,5	11062,5	2765,6	2765,6
15	0,0	0,0	2765,6	2765,6
17,5	0,0	0,0	0,0	0,0
20	2765,6	2765,6	0,0	0,0
22,5	0,0	0,0	0,0	0,0
25	2765,6	2765,6	0,0	0,0
27,5	0,0	0,0	0,0	0,0
30	0,0	0,0	0,0	0,0
32,5	0,0	0,0	0,0	0,0

Number of cycles (projected at the horizon) = f(coolant temperature, temperature range class)

Calculation of equivalent stress

Based on the definition of the test conditions, for example:

Tmin Coolant = -30°C, Tmax Coolant = 65°C giving ΔTe = 95°C

For each of the 4 cooling profiles, the number of equivalent thermal cycles Ne is:

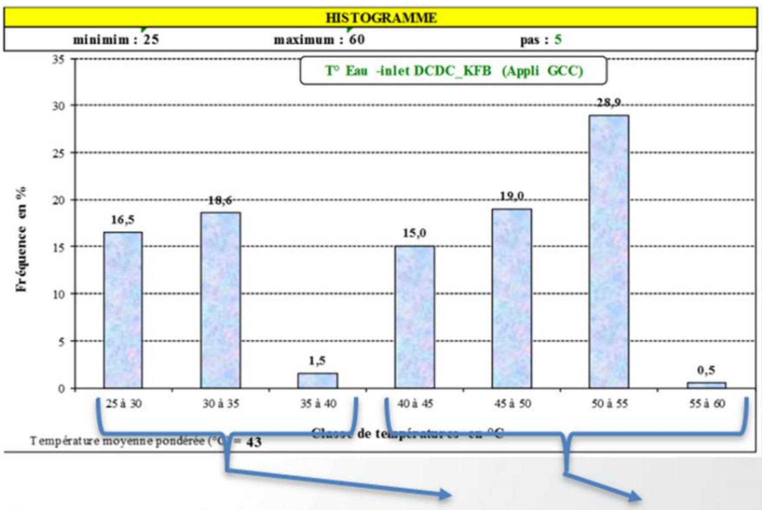
$$N_e = \sum N_1 \left(\frac{\Delta T_1}{\Delta T_e}\right)^n$$

Table 39: Number of equivalent cycles of the customer 63%

	Ne			
ΔTe(°K)	Tcoolant 1	Tcoolant 2	Tcoolant 3	Tcoolant 4
95	57,5	57,9	72,5	72,9
	14,5	14,1	3,6	3,6
	17,7	15,0	0,0	0,0
	22,6	16,1	3,2	3,2
	25,2	25,2	6,3	6,3
	0,0	0,0	10,9	10,9
	0,0	0,0	0,0	0,0
	25,8	25,8	0,0	0,0
	0,0	0,0	0,0	0,0
	50,4	50,4	0,0	0,0
	0,0	0,0	0,0	0,0
	0,0	0,0	0,0	0,0
	0,0	0,0	0,0	0,0
	0,0	0,0	0,0	0,0
	170,1	170,1	0,0	0,0
	0,0	0,0	0,0	0,0

Number of equivalent cycles (projected at the horizon) = f(coolant temperature)

From the considered geographical profile and its temperature histogram, each of the number of cycles *Ne* calculated previously for each of the 4 cooling profiles is multiplied by the ratio of the simplified distribution resulting from this histogram.



Tcoolant 1 (-20°C)	Tcoolant 2 (0°C)	Tcoolant3 (20°C)	Tcoolant4 (40°C)
0	0	37%	63%

Figure 72: Grouped coolant temperature histogram.

It can be observed that for the 4 profiles of the cooling circuit, only 2 are relevant for the calculation of the equivalent cumulative stress. The ECS is the result of the sum of the equivalent cycles resulting from this distribution.

In this example, based on the **mission profile of the cooling circuit,** the result of this processing gives an **equivalent cumulative stress (Ne)** of 97 cycles for a ΔTe of 95°C.

Parking mode stress phases

In the parking phase, the PEU are subject to the diurnal cycles like all vehicles.

The air and water after 2 to 3 hours in the parking phase are then at more or less the same temperature and then vary according to the outside temperature, the sunshine and the location of the cooling circuit and the PEU.

Acquisition of input data

Construction of the **environmental mission profile in passive mode** corresponding to the phases of inactivity of the complete vehicle.

Considering that the temperature difference between day and night throughout the year is on average 10°C.

If we take daily min-max records throughout the year over different regions of the planet, the result is oin the same order of magnitude.

Data processing

Over a 20-year life period, the number of diurnal cycles is 20x365.25, i.e. 7305 thermal cycles of 10°C amplitude.

Calculation of equivalent stress

The equivalent number of cycles is:

$$N_e = \sum N_1 \left(\frac{\Delta T_1}{\Delta T_e}\right)^n$$

We then obtain an **equivalent cumulative stress** (Ne) of 9 cycles for a ΔTe = 95°C.

Methodology to define the test plan.

The constraint is converted into a deterministic equivalent stress.

The methodology used to design the test plan (zero-failure truncated tests) is described in Chapter 5.4.3.1 of the guide.

$\tau = T * \left[\frac{ln(1-c)}{N*ln(1-P_f)}\right]^{\frac{1}{\beta}}$	**1- P_f :** level of reliability to be demonstrated	**0.995**[11]
	C: Level of confidence	**0.9**[12]
	β: Weibull shape parameter	**3,6**
	N: number of parts to be tested	**3**

To apply this formula to our endurance test:

- The number of cycles (T) is the equivalent customer 63% stress. It was previously calculated to be reduced to an equivalent damage below a defined ΔTe for the test.

- The number of cycles (τ) is the number of test cycles that the parts must undergo without failure to demonstrate the requested test survival rate with the selected confidence level.

The 3 sources of thermal cycling stresses having been converted into an equivalent cumulative stress on the same reference base for the endurance test, i.e. ΔTe = 95°C, they can be summed.

	ECS
The activation mission profile	**7**
The mission profile of the cooling system	**97**
The environmental mission profile in passive mode	**9**
Total ECS	**113**

While the environmental mission profile in passive mode remains more or less the same (order of magnitude: 10 ECS at ΔTe=95°C), this is not true for the other 2.

The mission profile of the cooling system depends on the cooling strategy at the system level. The equivalent cumulative stress can vary significantly. In the example, it is preponderant.

As for the mission profile during activation, it depends on the design of the PEU. The equivalent cumulative stress can vary greatly.

We then obtain a cumulative number of **equivalent stresses for our example** of 113 cycles, corresponding to the value (T)

Applying the above formula, the number of test cycles is 457 for a ΔTe = 95°C (τ)

Definition of the elementary endurance profile in thermal cycling

To determine the total duration of the test, it is necessary to know the thermal inertia of the PEU. It is therefore necessary to perform a thermal characterization on a functional part. This characterization has three goals:

a. Characterize the efficiency coefficient of the water circuit in relation to air (in OFF or ON mode).
b. Check the maximum self-heating of the most critical components.
c. Characterize the thermal time constants in Off mode (with respect to the cooling circuit) and On mode (with respect to the power components).

[11] **1- Pf** comes from the product specification.
[12] **C, β and N** are given as example only.

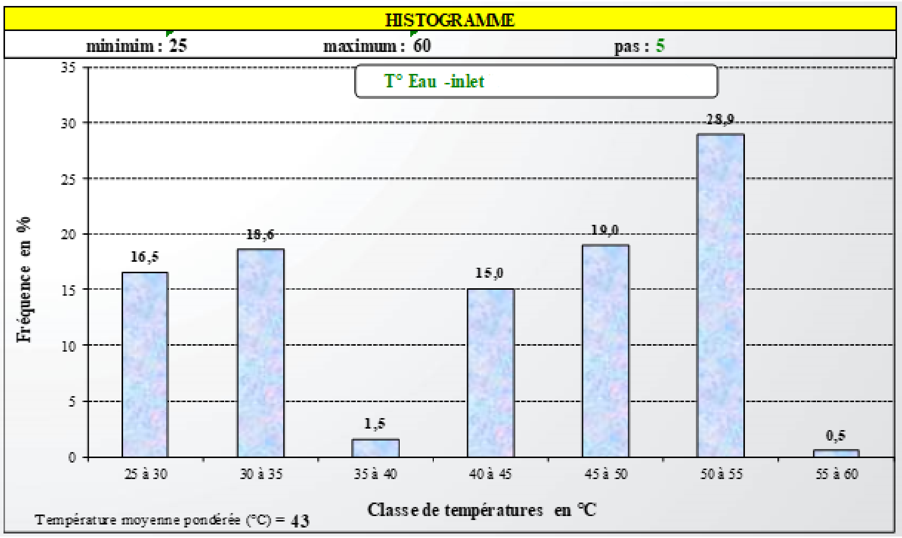


Figure 73: Example of characterization of a PEU composed of several PCBAs and electro-technical components (dot in red) under worst-case power conditions and under two air temperature conditions.

a. In this hundred-point measurement campaign, the measurements are grouped by function or by PCBA. For all measuring points, a coefficient of influence between water and air is calculated. If the dispersions are similar, the coefficients averages are made by group.

For a given thermocouple, the closer the blue and orange dots are, the better the component is cooled by water.

The cooling efficiency factor is defined as follows:

$$a = \frac{\Delta T_{air} - \Delta T_{part}}{\Delta T_{air}} = \frac{50 - \Delta T_{part}}{50}$$

We thus obtain:

a_avgPCA DRV = 92%

a_avgPCA PFC = 97%

a_avgPCA BOOST = 98%

a_avgPCA SMI = 99%

a_avgPCA INPUT = 84%

For a Tair variation of 50°C (test condition), the temperature of the PCA INPUT does not vary by more than 8°C, i.e. a cooling efficiency coefficient of 84%. The conclusion is that this PEU is very well cooled by water.

b. With the exception of the 4 measurement points on fuses (red circle) which rise above 120°C during this test with self-heating of more than 65°C, all the self-heating values read on the thermocouples are below 45°C with an average value over the whole of 28°C. The self-heating of the components is not critical.

c. The time constants are then characterized by PCBA and by high-volume power component to determine the optimal cycle time.

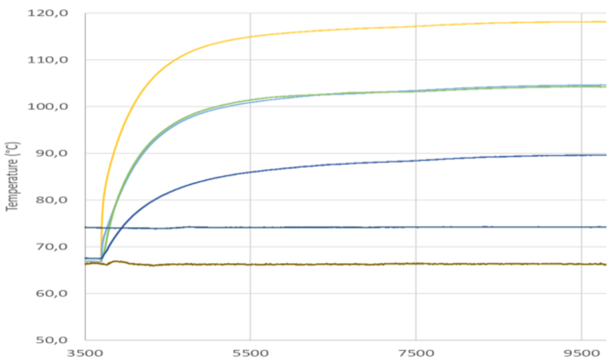


Figure 74: Example of thermal characterization on a PCBA of a PEU composed of several PCBA.

In this example, the time constant ζ in the ON phase of the 4 measurement points is 9 minutes. The thermal rise time from 10% to 90% is therefore 20 minutes (2.2 ζ).

To stabilize the components in temperature and to ensure good drift-free cycling, the non-operative relaxation phases must be longer than the thermal drop time from 90% to 10% at the component level.

A level with a minimum and maximum temperature of 25 minutes is selected. Then, you must consider the power capacities of the cooling system, which are not necessarily symmetrical in the ascent and in the descent.

Finally, if we want to reduce the number of test cycles and therefore the duration of the test, we must be able to increase the ΔTe. If possible, this leads to the following example.

When increasing the ΔTe, one must ensure not to exceed the AMR of the components, or even to keep a sufficient margin not to induce new failure modes that would not appear in the real life of the product.

Starting from 113 ECS cycles for a ΔTe of 95°C, and applying the Coffin-Manson law for a ΔTe of 110°C, we obtain 73 ECS cycles, i.e. a 35% reduction in the number of ECS cycles with an amplitude during the tests of -30°C to +80°C.

By applying the formula of test definition, the number of test cycles is then reduced from 457 to 295 test cycles.

On the profile below, a passage from -30 to +80°C or +80°C to -30°C in 22 minutes, with equipment to generate slopes of 5°C/min and 2 stabilization steps of 25 minutes leads to a complete cycle of 94 minutes (2x22 + 2x25 minutes).

The total duration of the test under these conditions is then ~19 days (295*1.56 hours). This is a very low value because it is not uncommon to have test durations of several months.

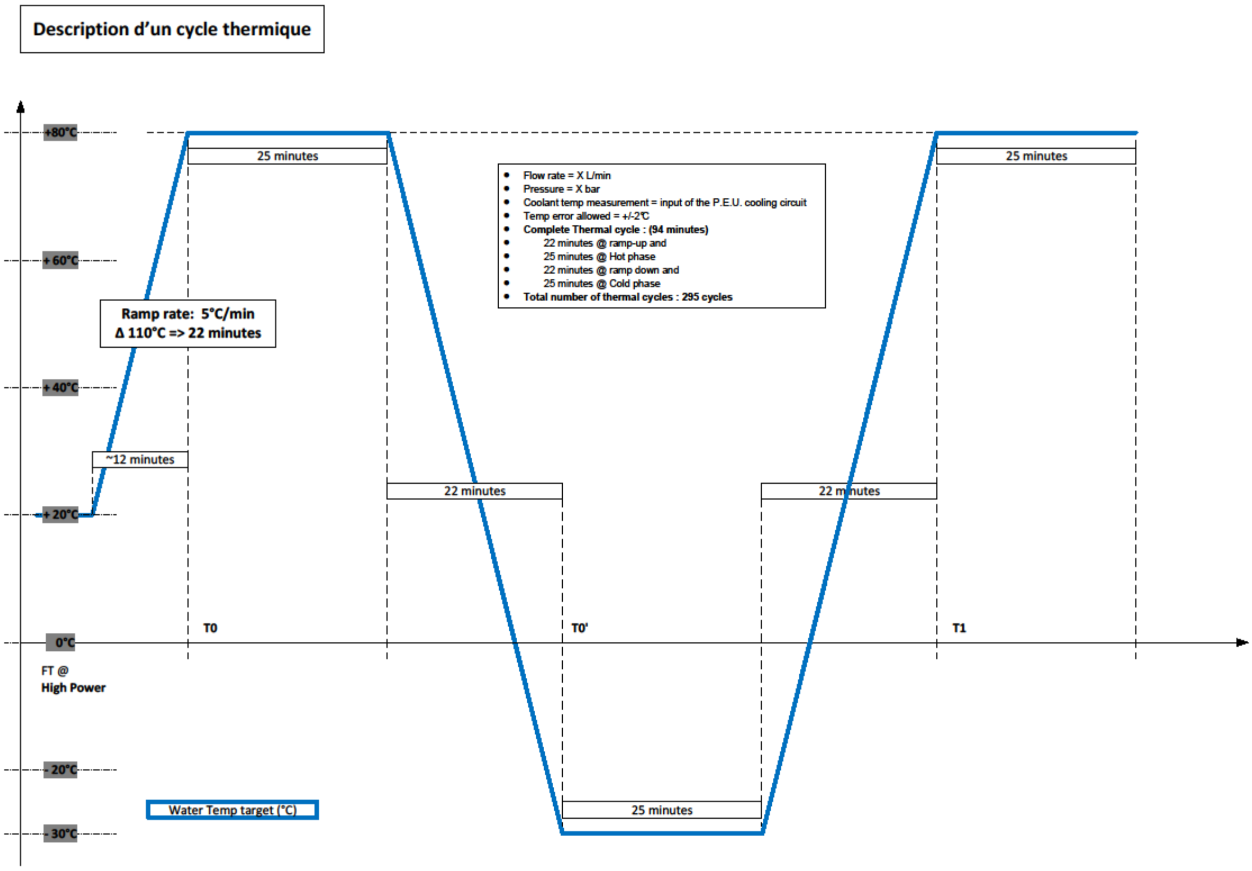


Figure 75: Elementary Test Profile

Practical sheet 15: Link and impact of the parameters of the objective on reliability validation

Objective

Reliability validation is based on the definition of an objective made of different parameters:

- Failure probability
- Reporting period: number of years and mileage (km)
- Confidence level

Partially expressing the objective would turn out to be incomplete.

This practical sheet proposes a study on the variability of these parameters regarding the reliability validation of a part.

Context and input

A supplier of load compartment cover received two specifications for two different manufacturers. The part is reserved for equal ranges of vehicles, so for the same clients. The mission profiles are considered to be the same for both manufacturers.

Feared event and definition of an objective (result of phase 1)

The feared event is the failure of the coiling spring of the load compartment cover. Each manufacturer defines a reliability objective according to the level of gravity.

Objective of manufacturer 1:

10,000 ppm in 10 years/180,000 km with a confidence level of 70%.

$$P_f = Prob(R \leq C) = 10^{-2}$$

Objective of manufacturer 2:

100,000 ppm in 10 years/180,000 km with a confidence level of 90%.

$$P_f = Prob(R \leq C) = 10^{-1}$$

R represents the distribution of Strength and C represents the distribution of Stress.

Physical mechanisms of failure and mission profiles (phases 2 and 3)

Fatigue and wear are the identified physical mechanisms of failure. A statistical law characterising the strength is defined for each physical mechanism (see ***Practical sheet 2***).

In the case of wear, only the **number of opening/closing cycles** is a damaging parameter.

In the case of fatigue, the **number of cycles** and the **level of pulling effort** are two damaging parameters linked through the Basquin relationship. It is then possible to set the level of effort and to calculate the corresponding number of cycles (and vice versa) (see ***Practical sheet 4***).

The gradient of Basquin is determined by the type of material (in the example, b=3).

The mission profiles regarding the number of opening/closing and the pulling effort are given in the table below.

Table 40: Mission Profile

Mission profiles	Type of distribution	Parameters of the distribution	Reporting period
Number of cover opening/closing	Logarithmic-normal	μln=8 σln=1	10 years / 180.000km
Pulling effort to open the cover (N)	Normal	μ=7 σ=1	-

<u>Calculation method for the scheduling of the validation plan (phase 4)</u>

The calculations made in this document are based on the probabilistic Stress-Strength analysis (see ***Practical sheet 7***).

The Stress-Strength analysis is used to characterise the optimum strength distribution $F_R(x)$ to meet the objective:

$$P_f = \int_{-\infty}^{+\infty} F_R(x) f_C(x, A) dx \rightarrow F_R(x) = \cdots$$

P_f: failure probability of the objective

F_R: distribution function of strength

f_C: stress probability density

A: reporting period of the objective

In the case of wear, only the number of cycles *n* is a damaging parameter. The optimum strength follows a Weibull distribution with a parameter β_{REX}=3 (experience feedback) and η_{obj}.

$$P_f = \int_{n_{min}}^{n_{max}} F_R(n, \beta_{REX}, \eta_{obj}) \times f_C(n, A) dn \rightarrow \eta_{obj} = \cdots$$

In the case of fatigue, the number of cycles *n* and the level of effort *s* are two damaging parameters. The optimum strength follows a normal distribution with parameters μ_{obj} and CVR_{REX} = 0.1 (variation coefficient from the experience feedback). The calculation is made with a set level of effort $s_{fixé}$, so with a corresponding number of cycles n_{eq} (see ***Practical sheet 4***).

$$n_{eq} = n \times \left(\frac{s}{s_{fixé}}\right)^b$$

n and *s* are respectively the random variables of the number of cycles and the level of effort; and *b* represents the gradient of Basquin.

$$P_f = \int_{n_{min}}^{n_{max}} \int_{s_{min}}^{s_{max}} F_R(n_{eq}, \mu_{obj}, CVR_{REX}) \times f_C(n_{eq}, A)\, ds\, dn \rightarrow \mu_{obj} = \cdots$$

Regarding wear and fatigue, the optimum calculated parameter of strength is used to determine the truncated test.

Calculation of the level of test τ in number of cycles for *Nb*=3 parts with a level of confidence c=70% or c=90%:

$$\tau_{usure} = F_R^{-1}\left(1-(1-c)^{\frac{1}{Nb}}, \beta_{REX}, \eta_{obj}\right)$$

$$\tau_{fatigue} = F_R^{-1}\left(1-(1-c)^{\frac{1}{Nb}}, \mu_{obj}, CVR_{REX}\right)$$

Concerning a well-defined test, the proportion of failures in test δc must be superior or equal to 0.1.

$$\delta_c = 1-(1-c)^{\frac{1}{Nb}}$$

Analysis of sensitivity of the failure probability on the validation plan

Study of the variability of the failure probability for the two main failure modes: fatigue and wear. Concerning fatigue, it has been decided to set the level of effort and to calculate the number of corresponding cycles. The validation test is a truncated test (no failure).

For each value of failure probability, the calculation of the optimum parameter of strength and the scheduling of the duration of the test are made. In the following tables, we chose to define a test with a fixed number of parts: Nb=3 parts with effort of 9N for the fatigue.

Table 41: Variability of failure probability

Failure probabilty	**Wear**			**Fatigue**		
	Strength: **Weibull** distribution Known param. (REX): β_R = 3	Definition of test: Set number of parts: Nb = 3		Strength: **Normal** distribution Known param. (REX): CVRR = 10%	Definition of test: Set number of parts: Nb = 3 Fixed effort S0 = 9N	
	Optimum parameter η_R	Testing time		Optimum parameter μ_R	Testing time	
		c=70% **Manufacturer 1**	c=90% **Manufacturer 2**		c=70% **Manufacturer 1**	c=90% **Manufacturer 2**
100.000 ppm	**14.600**	10.770 cycles	**13.368 cycles**	**5.498**	5.257 cycles	**5.548 cycles**
10.000 ppm	**48.390**	**35.694 cycles**	44.306 cycles	**17.358**	**16598 cycles**	17.515 cycles
1.000 ppm	122.826	90.597 cycles	112.455 cycles	40.230	38.467 cycles	40.592 cycles

For this example, the need to have a consistent value of the proportion of failures in test δc (close to 10%), does not allow to define the same test duration for the three values of failure probability.

The lower the probability of failure, the more severe the validation test plan is in number of cycles.

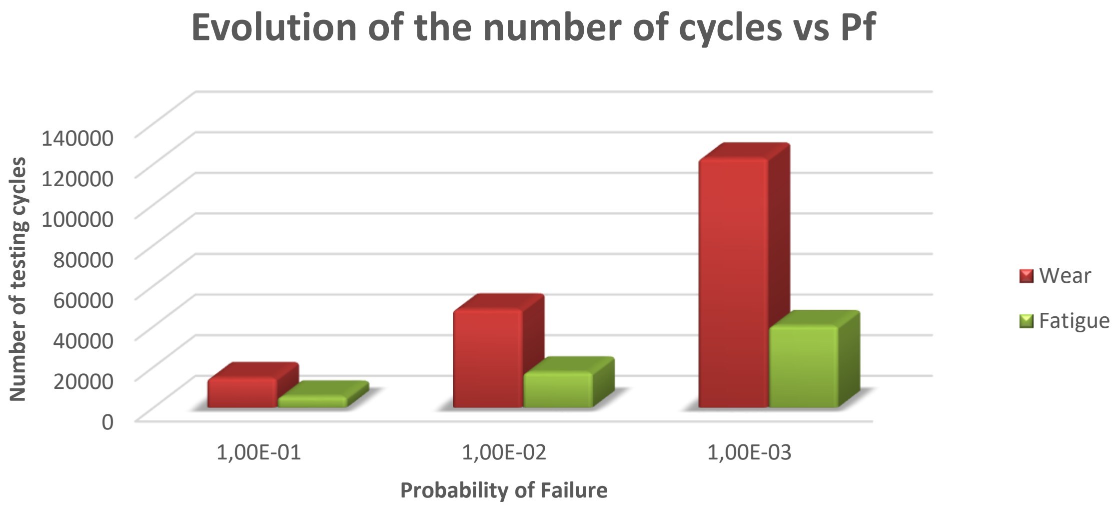


Figure 76: Evolution of the number of cycles as a function of the probability of failure for a confidence level c= 70% and 3 parts.

Analysis of the sensitivity of the confidence level on the validation plan

Study of the variability of the level of confidence for the two main failure modes: fatigue and wear. Concerning fatigue, it has been decided to set the level of effort and to calculate the number of corresponding cycles. The chosen validation test is a truncated test (no failure).

For each value of confidence level, the emphasis is put on the determination of the test. As in the previous chapter, it has been decided to define a test 1 (set number of parts *Nb=3 parts*) and a test 2 (set duration of test).

Table 42: Variability of the confidence level with P_f=10^{-2}

Confidence level	**Wear**			**Fatigue**		
	Strength: **Weibull** distribution Known param. (REX): β_R = 3	Definition of test: Set number of parts: Nb = 3	Definition of test: Testing time Ncycles = 30.000 cycles	Strength: **Normal** distribution Known param. (REX): CVRR = 10%	Definition of test: Set number of parts: Nb = 3	Definition of test: Testing time Ncycles = 30.000 cycles
	Optimum parameter η_R	Testing time	Nb parts	Optimum parameter μ_R	Testing time	Nb parts
70%	48.390	**35.694 cycles**	**6**	17.358	**16.598 cycles**	**8**
90%		44.306 cycles	10		17.515 cycles	15

Table 43: Variability of the confidence level with P_f=10^{-1}

Confidence level	Wear			Fatigue		
	Strength: **Weibull** distribution Known param. (REX): β_R = 3	Definition of test: Set number of parts: Nb = 3	Definition of test: Testing time Ncycles = 30.000 cycles	Strength: **Normal** distribution Known param. (REX): CVRR = 10%	Definition of test: Set number of parts: Nb = 3	Definition of test: Testing time Ncycles = 30.000 cycles
	Optimum parameter η_R	Testing time	Nb parts	Optimum parameter μ_R	Testing time	Nb parts
70%	14.600	10.770 cycles	4	5.498	5.257 cycles	6
90%		13.368 cycles	8		5.548 cycles	12

A high confidence level leads to an increase in test time and number of parts. But with a much smaller impact than the variation in failure probability over the number of cycles.

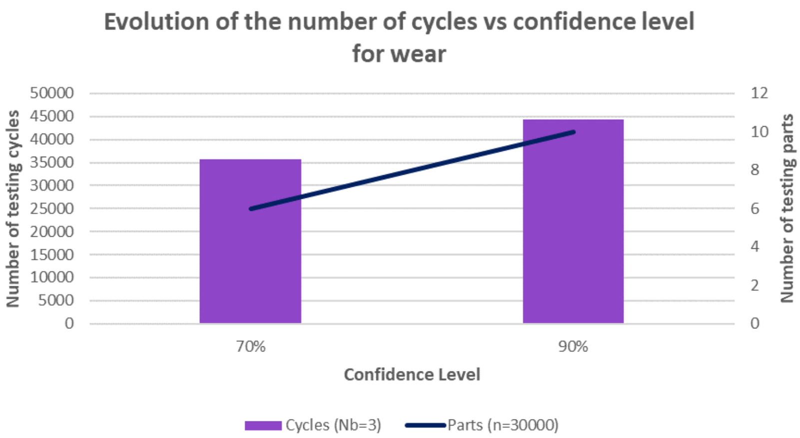


Figure 77: Evolution of the number of parts and cycles as a function of the confidence level for wear with a P_f=10^{-2}.

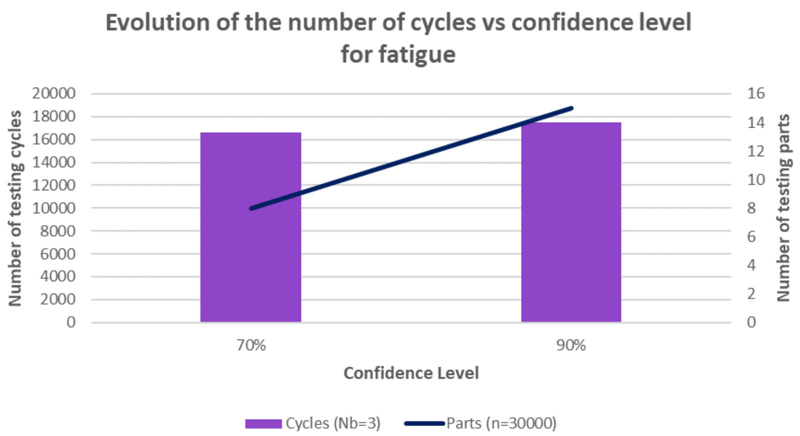


Figure 78: Evolution of the number of parts and cycles as a function of the confidence level for fatigue with a P_f=10^{-2}.

Analysis conclusions

The supplier of the load compartment cover defined two validation plans of the truncated tests (no failure) respecting the specifications of both manufacturers.

The supplier can make a unique test on its test benches suiting both manufacturers.

It is possible to use an identical number of parts and to continue with the number of cycles corresponding to the most demanding manufacturer.

The table below shows the values of number of cycles to be made in order to validate the objective of each manufacturer with the same number of parts used.

Table 44: Number of cycles for the same number of parts

Number of parts	Wear		Fatigue	
	Manufacturer 1 10.000 ppm ; c= 70%	Manufacturer 2 100.000 ppm ; c= 90%	Manufacturer 1 10.000 ppm ; c= 70%	Manufacturer 2 100.000 ppm ; c= 90%
3	35.694 cycles	13.368 cycles	16.598 cycles	5.548 cycles
7	26.911 cycles	10.079 cycles	15.618 cycles	5.179 cycles
10	23.895 cycles	8.949 cycles	15.261 cycles	5.047 cycles

NOTE 1: It is recommended to make the test with at least three parts to limit the probability of testing a defective part.

NOTE 2: Reducing the number of parts leads to a cost reduction for the tests. Reducing the number of cycles leads to the optimisation of the schedule regarding the downtime of the test benches.

NOTE 3: In this example, it is not possible to define a duration of cycles in test common to both manufacturers' objectives. With the objective of manufacturer 2, the parameter δ is lower than 0.1 for a number of cycles higher than 15,000.

NOTE 4: It is possible for the supplier to carry out a single test for the validation of the two objectives in fatigue and wear by taking the highest number of cycles for the same number of parts; at a force adapted to the number of cycles chosen (determined with the principle of fatigue equivalence).By taking into account the hypothesis that the number of opening/closing cycles is the only damaging parameter for wear, the test can actually be carried out at any force.

Table 45: Effort equivalent to the envelope number of cycles

Number of parts	Envelope number of cycles with an effort of 9N for wear & fatigue		Test conditions adapted for both manufacturers for wear & fatigue
	Manufacturer 1 10.000 ppm ; c=70%	Manufacturer 2 100.000 ppm ; c=90%	
3	35.694 cycles	13.368 cycles	35.694 cycles with an effort of 6,97 N
7	26.911 cycles	10.079 cycles	26.911 cycles with an effort of 7,66 N
10	23.895 cycles	8.949 cycles	23.895 cycles with an effort of 7,97 N

The figures below illustrate the different possible scenarios for carrying out the test.

Figure 79 illustrates the planning aspect. It shows that, for the specifications requested by the two manufacturers, the number of cycles decreases with the increase in the number of parts to finally reach a saturation level. Therefore, for the test, considering that the parts are tested simultaneously, the gain is negligible in terms of bench occupancy from 7 parts.

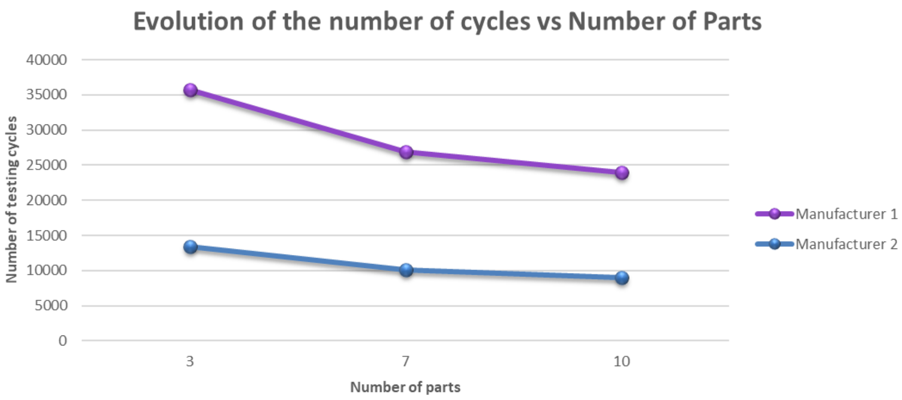


Figure 79: Evolution of the number of cycles according to the number of parts for each manufacturer.

Figure 80 illustrates the cost aspect. It shows that testing 3 parts separately would cost less than testing 10 parts separately based on the time the testing machine is occupied.

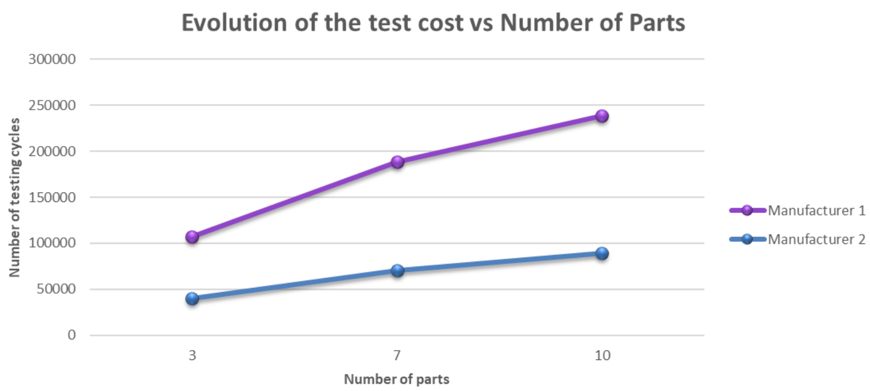


Figure 80: Evolution of the product of the number of cycles and the number of parts as a function of the number of parts for each manufacturer.

It is possible for the supplier of load compartment covers to make the following economic assessment in order to perform a unique test suitable for both manufacturers and to adjust the cost of this test: cost A of the number of parts vs. cost B of benches downtime.

If A > B → specifications of both manufacturers are validated for both failure modes with 3 parts at 35,694 cycles and an effort of 7 N without any failure.

If B > A → specifications of both manufacturers are validated for both failure modes with 10 parts at 23,895 cycles and an effort of 8 N without any failure when the bench is sized to performed tests at the same time.

Conclusion

Several conclusions can be drawn from this practical sheet:

- The failure probability is the most important parameter in terms of severity of the test. A division by 10 of this probability induces approximately a multiplication by 3 of the number of cycles.
- The failure probability must be given with an associated reference period and confidence level.
- The confidence level also has an influence on the cost of a validation plan.
- This reporting period and the failure probability must be adapted to the gravity of the feared event (see Farmer Curve)

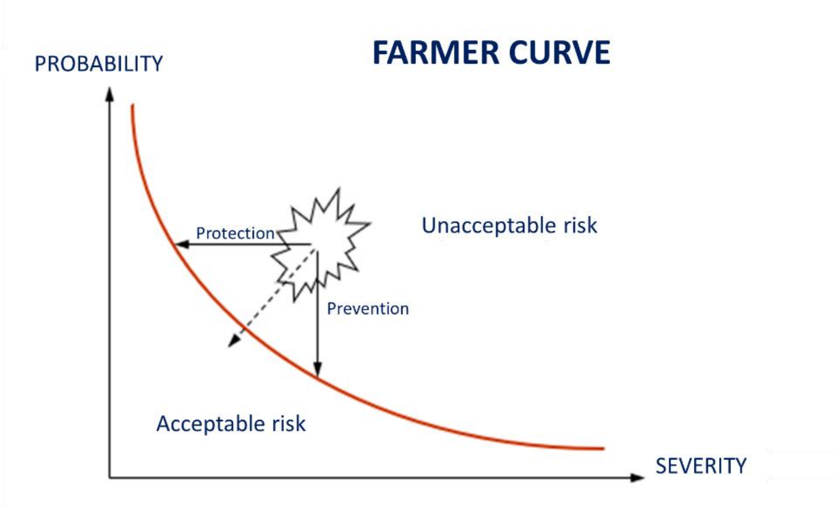


Figure 81: FARMER Curve.

Annex 1: Lists of figures and tables

LIST OF FIGURES

LISTS OF TABLES

Annex 2: References

Theme 1: **Prevention of early failures**

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<u>Theme 6:</u> Fatigue

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